Answer Keys

8-1 Additional Vocabulary Support

Adding and Subtracting Polynomials

Concept List

binomial	constant	cubic
degree	fourth degree	linear
monomial	quadratic	trinomial

Choose the concept or concepts from the list above that best represent the item in each box.

1. $2x^3 + 5$ cubic/binomial	**2.** $5x + 4x^2$ binomial/quadratic	**3.** 8 constant/monomial
4. 9 constant/monomial	**5.** $3x^2 + 6x + 4$ quadratic/trinomial	**6.** $3x^2 + 6x$ quadratic/binomial
7. $4x^4 + 6x^3 + 2x^2$ fourth degree/trinomial	**8.** $7x^2 + x$ ↑ 2 degree	**9.** $5x^4$ fourth degree/monomial

8-1 Reteaching

Adding and Subtracting Polynomials

You can add and subtract polynomials by lining up like terms and then adding or subtracting each part separately.

Problem

What is the simplified form of $(3x^2 - 4x + 5) + (5x^2 + 2x - 8)$?

Write the problem vertically, lining up the like terms. Then add each pair of like terms.

$$\begin{array}{r} 3x^2 - 4x + 5 \\ +\ 5x^2 + 2x - 8 \\ \hline \end{array}$$

Solve Add the x^2 terms. Add the x terms. Add the constant terms.

$3x^2 + 5x^2 = 8x^2$ $\quad -4x + 2x = -2x$ $\quad 5 + (-8) = -3$

$$\begin{array}{r} 3x^2 - 4x + 5 \\ +\ 5x^2 + 2x - 8 \\ \hline 8x^2 - 2x - 3 \end{array}$$

Add the sums.

Check Check your solution using subtraction.

$8x^2 - 5x^2 = 3x^2$ $\quad -2x - 2x = -4x$ $\quad -3 - (-8) = 5$

Solution: $(3x^2 - 4x + 5) + (5x^2 + 2x - 8) = 8x^2 - 2x - 3$

Exercises

Simplify.

1. $\begin{array}{r} 5b^2 + 3b \\ +\ 2b^2 - 5b \\ \hline \end{array}$

$7b^2 - 2b$

2. $\begin{array}{r} 3c^2 + 3c \\ +\ 4c^2 + 2c \\ \hline \end{array}$

$7c^2 + 5c$

3. $\begin{array}{r} 4d^2 - 3d + 6 \\ +\ 2d^2 + 5d - 3 \\ \hline \end{array}$

$6d^2 + 2d + 3$

4. $\begin{array}{r} -3e^2 - 5e + 2 \\ +\ e^2 + 2e - 7 \\ \hline \end{array}$

$-2e^2 - 3e - 5$

5. $\begin{array}{r} 4f^3 + 2f^2 + 5f \\ +\ 2f^3 - 4f^2 - 3f \\ \hline \end{array}$

$6f^3 - 2f^2 + 2f$

6. $\begin{array}{r} 5g^3 - 2g^2 + 3g \\ +\ 2g^3 + 5g^2 - 2g \\ \hline \end{array}$

$7g^3 + 3g^2 + g$

7. $(3h^2 + 5) + (-5h^2 - 3)$

$-2h^2 + 2$

8. $(2j^2 + 4j - 6) + (4j^2 - 3j - 3)$

$6j^2 + j - 9$

8-1 Reteaching (continued)

Adding and Subtracting Polynomials

To subtract polynomials, follow the same steps as in addition.

Problem

What is the simplified form of $(6x^3 + 4x^2 - 3x) - (2x^3 + 3x^2 - 5x)$?

Write the problem vertically, lining up the like terms. Then subtract each pair of like terms.

$$\begin{array}{r} 6x^3 + 4x^2 - 3x \\ -\ (2x^3 + 3x^2 - 5x) \\ \hline \end{array}$$

Solve

Subtract the x^3 terms.

$6x^3 - 2x^3 = 4x^3$

Subtract the x^2 terms.

$4x^2 - 3x^2 = x^2$

Subtract the x terms.

$-3x - (-5x) = 2x$

$$\begin{array}{r} 6x^3 + 4x^2 - 3x \\ -\ (2x^3 + 3x^2 - 5x) \\ \hline 4x^3 + x^2 + 2x \end{array}$$

Add the differences.

Check Check your solution using subtraction.

$4x^3 + 2x^3 = 6x^3$ $\quad x^2 + 3x^2 = 4x^2$ $\quad 2x + (-5x) = -3x$

Solution: $(6x^3 + 4x^2 - 3x) - (2x^3 + 3x^2 - 5x) = 4x^3 + x^2 + 2x$

Exercises

Simplify.

9. $$\begin{array}{r} 4k^2 + 5k \\ -\ (3k^2 + 2k) \\ \hline \end{array}$$
$k^2 + 3k$

10. $$\begin{array}{r} 5m^2 - 4m \\ -\ (2m^2 + 3m) \\ \hline \end{array}$$
$3m^2 - 7m$

11. $$\begin{array}{r} 7n^2 + 4n + 9 \\ -\ (4n^2 + 3n + 5) \\ \hline \end{array}$$
$3n^2 + n + 4$

12. $$\begin{array}{r} 5p^2 + 6p + 4 \\ -\ (7p^2 + 4p + 8) \\ \hline \end{array}$$
$-2p^2 + 2p - 4$

13. $$\begin{array}{r} 3q^3 + 2q^2 + 7q \\ -\ (6q^3 - 4q^2 - 5q) \\ \hline \end{array}$$
$-3q^3 + 6q^2 + 12q$

14. $$\begin{array}{r} 2r^3 - 2r^2 + 5r \\ -\ (4r^3 + 5r^2 + 3r) \\ \hline \end{array}$$
$-2r^3 - 7r^2 + 2r$

15. $(6s^2 - 5s) - (-2s^2 + 3s)$

$8s^2 - 8s$

16. $(3w^2 + 6w - 5) - (5w^2 - 4w + 2)$

$-2w^2 + 10w - 7$

8-1 Puzzle: Polynomial Search

Adding and Subtracting Polynomials

You will find the answers to the problems below somewhere in the grid. An answer may appear straight across, straight down, or on a diagonal. Circle each problem and answer you find. The first one has been done for you.

a. $4x^2 + 7x + 9 - (2x^2 + 10x + 4)$ **b.** $x^5 - x^3 - x^2 + 3 + (x^4 - 2x^3 - 2x^2 + 4x - 3)$

c. $-2x^4 - x + 3 + (8x^5 + 8x^4 - 3x + 1)$ **d.** $7x^4 - 2x^2 + 5x + 4 - (x^5 - 2x^4 - 6x^3 + 4x)$

e. $6x^5 - x^4 + 5x^3 + (3x^5 + 8x^4 - 6x^2 + 3)$ **f.** $x^5 - 3x^3 + 6x^2 + x - (x^5 + 5x^3 - x^2 + x)$

g. $-2x^4 - x^2 + 5 - (2x^5 + 3x^4 - 6x^2 + x)$ **h.** $3x^5 - 4x^4 + 3x^2 + 7 + (-4x^4 - 4x^2 - x + 1)$

i. $2x^4 + x^3 + x^2 + 6 - (2x^4 + 5x^3 - x^2 + 3x + 7)$

j. $-x^5 - x^2 + 3x - 1 - (2x^5 + x^3 + x^2 + 5x - 4)$

k. $-x^4 + 4x^3 + 6x^2 + 1 + (2x^4 - 4x^3 - x^2 + 3x + 3)$

l. $-x^5 - x^4 + x^3 + x^2 + 2 - (x^4 - 4x^3 + 2x^2 + x - 3)$

$2x^2$	$9x^2$	5	$2x^3$	x^4	$5x^2$	$3x$	4	$2x^3$	$3x^2$	5	$-x^2$
5	$-3x$	$-7x^3$	$2x$	$2x^6$	$5x^9$	$2x^3$	x^2	$-x^9$	$3x^6$	$-2x^5$	5
$-7x^3$	$2x$	5	$5x^9$	$7x^2$	$3x^6$	$2x^3$	$-x^7$	$-x^5$	$7x^2$	$-5x^4$	x^2
$3x^6$	$5x^2$	$9x^2$	$-8x^3$	$-4x^7$	-1	$2x$	-1	$-2x^4$	$2x$	$5x^2$	-1
$-9x^3$	$9x^5$	$7x^4$	$5x^3$	$-6x^2$	3	$2x^7$	x^2	$5x^3$	$5x^9$	$-x$	$2x$
$3x^5$	$-8x^4$	$-x^2$	$-x$	8	$2x^6$	$-x^6$	-1	$-x^2$	$-3x^7$	5	$8x^2$
$2x$	x^7	$-5x^3$	$5x^7$	4	$2x^2$	$-4x^3$	$2x^5$	$-x$	$-x^2$	$-x^2$	$2x^5$
5	$8x^2$	$-x^2$	$-4x$	$5x^9$	$-9x^3$	$2x^2$	$3x^7$	5	$2x^6$	$-5x^7$	3
$2x^3$	$2x^9$	$6x^4$	$-3x^7$	-1	x^7	$-3x$	$5x^9$	5	6	$-2x$	5
$2x^6$	$8x^5$	1	$-9x^7$	$8x^2$	$2x$	-1	$2x$	$-9x^3$	$-2x^2$	$3x^9$	$3x^6$
$-x^5$	$9x^4$	$6x^3$	$-2x^2$	x	4	$-5x^3$	2	$-x^3$	1	$-9x^3$	$3x^7$
$8x^2$	x^5	x^4	$-3x^3$	$-3x^2$	$4x$	3	$-3x^5$	$2x^2$	$8x^2$	$3x^6$	$2x^5$

8-1 Think About a Plan

Adding and Subtracting Polynomials

Geometry The perimeter of a trapezoid is $39a - 7$. Three sides have the following lengths: $9a$, $5a + 1$, and $17a - 6$. What is the length of the fourth side?

Understanding the Problem

1. What is the perimeter of the trapezoid? $39a - 7$

2. What are the lengths of the sides you are given? $9a$, $5a + 1$, $17a - 6$

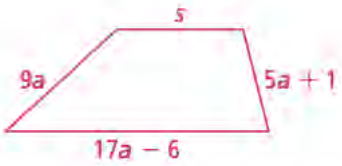
3. How many sides does a trapezoid have? 4 sides

4. How do you find the perimeter of a trapezoid? add the side lengths

5. What is the problem asking you to determine? the length of the fourth side

Planning the Solution

6. Draw a diagram of the trapezoid and label the information you know.



7. Write an equation that can be used to determine the length of the fourth side.

$s = (39a - 7) - (9a + 5a + 1 + 17a - 6)$

Getting an Answer

8. Solve your equation to find the length of the fourth side of the trapezoid.

$8a - 2$

8-1 Practice

Form G

Adding and Subtracting Polynomials

Find the degree of each monomial.

1. $2b^2c^2$ **4**
2. $5x$ **1**
3. $7y^5$ **5**
4. $19ab$ **2**
5. 12 **0**
6. $\frac{1}{2}z^2$ **2**
7. t **1**
8. $4d^4e$ **5**

Simplify.

9. $2a^3b + 4a^3b$ **$6a^3b$**
10. $5x^3 - 4x^3$ **x^3**
11. $3m^6n^3 - 5m^6n^3$ **$-2m^6n^3$**
12. $-6ab + 3ab$ **$-3ab$**
13. $4c^2d^6 - 7c^2d^6$ **$-3c^2d^6$**
14. $315x^2 - 30x^2$ **$285x^2$**

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.

15. $15x - x^3 + 3$
$-x^3 + 15x + 3$; cubic trinomial
16. $5x + 2x^2 - x + 3x^4$
$3x^4 + 2x^2 + 4x$; fourth degree trinomial
17. $9x^3$
$9x^3$; cubic monomial
18. $7b^2 + 4b$
$7b^2 + 4b$; quadratic binomial
19. $-3x^2 + 11 + 10x$
$-3x^2 + 10x + 11$; quadratic trinomial
20. $12t^2 + 1 - 3x + 8 - 2x$
$12t^2 - 5x + 9$; quadratic trinomial

Simplify.

21. $$\begin{array}{r} 8z - 12 \\ +\ 6z + 9 \\ \hline \end{array}$$
$14z - 3$

22. $$\begin{array}{r} 9x^3 + 3 \\ +\ 4x^3 + 7 \\ \hline \end{array}$$
$13x^3 + 10$

23. $$\begin{array}{r} 6j^2 - 2j + 5 \\ +\ 3j^2 + 4j - 6 \\ \hline \end{array}$$
$9j^2 + 2j - 1$

24. $(3k^2 + 5) + (16x^2 + 7)$
$3k^2 + 16x^2 + 12$

25. $(g^4 - 4g^2 + 11) + (-g^3 + 8g)$
$g^4 - g^3 - 4g^2 + 8g + 11$

26. A local deli kept track of the sandwiches it sold for three months. The polynomials below model the number of sandwiches sold, where s represents days.

Ham and Cheese: $4s^3 - 28s^2 + 33s + 250$
Pastrami: $-7.4s^2 + 32s + 180$

Write a polynomial that models the total number of these sandwiches that were sold. **$4s^3 - 35.4s^2 + 65s + 430$**

8-1 Practice (continued)

Form G

Adding and Subtracting Polynomials

Simplify.

27. $\begin{array}{r} 11n - 4 \\ -\ (5n + 2) \\ \hline 6n - 6 \end{array}$

28. $\begin{array}{r} 7x^4 + 9 \\ -\ (8x^4 + 2) \\ \hline -x^4 + 7 \end{array}$

29. $\begin{array}{r} 3d^2 + 8d - 2 \\ -\ (2d^2 - 7d + 6) \\ \hline d^2 + 15d - 8 \end{array}$

30. $(28e^3 + 3e^2) + (19e^3 + e^2)$

$47e^3 + 4e^2$

31. $(-12h^4 + h) - (-6h^4 + 3h^2 - 4h)$

$-6h^4 - 3h^2 + 5h$

32. A small town wants to compare the number of students enrolled in public and private schools. The polynomials below show the enrollment for each:

Public School: $-19c^2 + 980c + 48{,}989$

Private School: $40c + 4046$

Write a polynomial for how many more students are enrolled in public school than private school. $-19c^2 + 940c + 44{,}943$

Simplify. Write each answer in standard form.

33. $(3a^2 + a + 5) - (2a - 5)$

$3a^2 - a + 10$

34. $(6d - 10d^3 + 3d^2) - (5d^3 + 3d - 4)$

$-15d^3 + 3d^2 + 3d + 4$

35. $(-4s^3 + 2s - 3) + (-2s^2 + s + 7)$

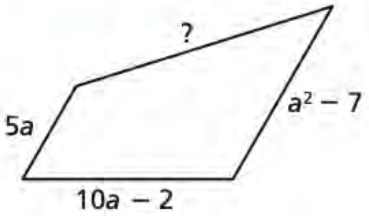
$-4s^3 - 2s^2 + 3s + 4$

36. $(8p^3 - 6p + 2p^2) + (9p^2 - 5p - 11)$

$8p^3 + 11p^2 - 11p - 11$

37. The fence around a quadrilateral-shaped pasture is $3a^2 + 15a + 9$ long. Three sides of the fence have the following lengths: $5a$, $10a - 2$, $a^2 - 7$. What is the length of the fourth side of the fence?

$2a^2 + 18$



38. Error Analysis Describe and correct the error in simplifying the sum shown at the right.

$$\begin{array}{r} 6x^3 + 4x - 10 \\ +\ (-3x^2 + 2x + 8) \\ \hline 3x^3 + 6x - 2 \end{array}$$

two unlike terms, $6x^3$ and $-3x^2$, were added; $6x^3 - 3x^2 + 6x - 2$

39. Open-Ended Write three different examples of the sum of a quadratic trinomial and a cubic monomial.

Answers may vary. Sample: $(x^2 + 2x + 1) + x^3$; $(2x^2 + 5x + 6) + 3x^3$; $(r^2 + r + 1) + 8r^3$

8-1 Practice

Form K

Adding and Subtracting Polynomials

Find the degree of each monomial.

1. $3s^3t^3$ **6**

2. $3n$ **1**

3. $5xy$ **2**

4. 7 **0**

5. $\frac{1}{4}k^5$ **5**

6. d **1**

Simplify.

7. $3mn^4 + 6mn^4$ $9mn^4$

8. $12g^2 - 7g^2$ $5g^2$

9. $-11c^4d + 12c^4d$ c^4d

10. $42z^3 - 15z^3$ $27z^3$

Write each polynomial in standard form. Then name each polynomial based on its degree and number of terms.

11. $7a + 4 - a^2$

$-a^2 + 7a + 4$; quadratic trinomial

12. $5b^2 + 2n$

$5b^2 + 2n$; quadratic binomial

13. $-11d^4$

$-11d^4$; 4th degree monomial

14. $2x^3 - 9 + 2x + 8 - 4x$

$2x^3 - 2x - 1$; cubic trinomial

15. A pizza shop owner is monitoring the amount of cheese he uses each week. The polynomials below model the pounds of cheese ordered in the past, where p represents pounds.

Mozzarella: $3p^3 - 6p^2 + 14p + 125$

Cheddar: $12.5p^2 + 18p + 75$

Write a polynomial that models the total number of pounds of cheese that were ordered.

$3p^3 + 6.5p^2 + 32p + 200$

8-1 Practice (continued)

Form K

Adding and Subtracting Polynomials

Simplify.

16. $\begin{array}{r} 3r + 5 \\ + \ 7r + 3 \\ \hline \end{array}$

$10r + 8$

17. $(t^4 - 4t^2 + 9) + (-t^3 + 3t)$

$t^4 - t^3 - 4t^2 + 3t + 9$

18. $\begin{array}{r} 7b^2 + 6 \\ + \ 4b^2 + 5 \\ \hline \end{array}$

$11b^2 + 11$

19. $\begin{array}{r} 4z + 7 \\ - \ (6z + 1) \\ \hline \end{array}$

$-2z + 6$

20. $(-6k^3 + 3k) - (-5k^3 + 3k^2 - 8k)$

$-k^3 - 3k^2 + 11k$

21. $\begin{array}{r} 3p^4 + 1 \\ - \ (9p^4 + 5) \\ \hline \end{array}$

$-6p^4 - 4$

22. A city wants to compare the number of people who own their own home and who rent their home. The polynomials below show expressions for each. In each polynomial, $p = 0$ corresponds to the first year.

Own: $4p^2 + 360p + 22{,}178$

Rent: $6p^2 + 125p + 5286$

Write a polynomial for how many more own their home than rent their home.

$-2p^2 + 235p + 16{,}892$

23. The wallpaper border that runs all the way around a room is $5f^2 + 19f + 11$ long. Three sides of the room have the following lengths of border: $6f$, $5f - 7$, $2f^2 + 2$. What is the length of the fourth side of the room?

$3f^2 + 8f + 16$

24. Open-Ended Write two different polynomials with a difference of $-3x^2 + 5x - 7$.

Answers may vary. Sample: $(-1x^2 + 6x - 4) - (2x^2 + x + 3)$ and $(-4x^2 + 7x - 5) - (-x^2 + 2x + 2)$

8-1 Enrichment

Adding and Subtracting Polynomials

Packing boxes and packing sheets in different sizes are given by the expressions below. To find the number of packing boxes and sheets that will fit in a larger shipping box, add or subtract the polynomials. Tell the total number of boxes and sheets. Then tell how many medium and large boxes and sheets you could fit into the shipping box. The first one has been started for you.

Boxes	a^3 = small box	b^3 = medium box	c^3 = large box
	6 small boxes = 1 medium box	4 medium boxes = 1 large box	
Sheets	a^2 = small sheet	b^2 = medium sheet	c^2 = large sheet
	4 small sheets = 1 medium sheet	8 medium sheets = 1 large sheet	

1. $7a^3 + 5b^3 + 5a^3 - 3b^3 = 12a^3 - 2b^3$

= 12 small boxes and 2 medium boxes

= 4 medium boxes

= 1 large box(es)

2. $6a^2 + 3b^2 - 8c^2 + 12b^2 - 2a^2 + 10c^2 =$ $4a^2 + 15b^2 + 2c^2$

= 4 small sheets, 15 medium sheets, and 2 large sheets

= 16 medium sheets and 2 large sheets

= 4 large sheets

3. $(8a^3 - 3b^3 + 6c^3) - (2a^3 - 14b^3 + 2c^3) =$ $6a^3 + 11b^3 + 4c^3$

= 6 small boxes, 11 medium boxes, and 4 large boxes

= 12 medium boxes and 4 large boxes

= 7 large boxes

4. $(15c^2 + 12a^3 - 9b^2) + (-14c^2 + 6a^3 + 5b^3 + 25b^2) =$ 18 small boxes,

= 5 medium boxes, 16 medium sheets, and 1 large sheet $18a^3 + 5b^3 + 16b^2 + c^2$

= 8 medium boxes 16 medium sheets, and 1 large sheets

= 2 large boxes and 3 large sheets

8-2 Additional Vocabulary Support

Multiplying and Factoring

There are two sets of note cards below that show how Brittany factors the polynomial $5x^5 + 15x^3 + 4x^2$. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

Think Cards

Factor each term of the polynomial.

Simplify.

Find the GCF of the three terms.

Factor out the GCF from each term.

Write Cards

$x^2(5x^3 + 15x + 4)$

The GCF is $x \cdot x$, or x^2.

$5x^5 + 15x^3 + 4x^2 =$
$x^2(5x^3) + x^2(15x) + x^2(4)$

$5x^5 = 5 \cdot x \cdot x \cdot x \cdot x \cdot x$
$15x^3 = 3 \cdot 5 \cdot x \cdot x \cdot x$
$4x^2 = 2 \cdot 2 \cdot x \cdot x$

Think

First, she should factor each term of the polynomial.

Second, she should find the GCF of the three terms.

Next, she should factor out the GCF from each term. Then factor it out of the polynomial.

Finally, she should simplify.

Write

Step 1 $5x^5 = 5 \cdot x \cdot x \cdot x \cdot x \cdot x$
$15x^3 = 3 \cdot 5 \cdot x \cdot x \cdot x$
$4x^2 = 2 \cdot 2 \cdot x \cdot x$

Step 2 The GCF is $x \cdot x$, or x^2.

Step 3 $5x^5 + 15x^3 + 4x^2 =$
$x^2(5x^3) + x^2(15) + x^2(4)$

Step 4 $x^2(5x^3 + 15x + 4)$

8-2 Reteaching

Multiplying and Factoring

You can multiply a monomial and a trinomial by solving simpler problems. You can use the Distributive Property to make three simpler multiplication problems.

Problem

What is the simplified form of $3x(2x^2 + 4x - 1)$?

Use the Distributive Property to rewrite the problem as three separate multiplication problems.

$$3x(2x^2 + 4x - 1) = (3x \cdot 2x^2) + (3x \cdot 4x) + (3x \cdot (-1))$$

Remember that when you multiply same-base terms containing exponents, you add the exponents.

Solve	$3x \cdot 2x^2 = 6x^3$	Multiply inside the first pair of parentheses.
	$3x \cdot 4x = 12x^2$	Multiply inside the second pair of parentheses.
	$3x \cdot (-1) = -3x$	Multiply inside the third pair of parentheses.
	$6x^3 + 12x^2 - 3x$	Add the products.
Check	$6x^3 \div 2x^2 = 3x$	Check your solution using division.
	$12x^2 \div 4x = 3x$	
	$-3x \div (-1) = 3x$	

Solution: $3x(2x^2 + 4x - 1) = 6x^3 + 12x^2 - 3x$

Exercises

Simplify each product.

1. $4x(2x - 7)$
   $8x^2 - 28x$
2. $3y(3y + 4)$
   $9y^2 + 12y$
3. $2z^2(2z - 3)$
   $4z^3 - 6z^2$
4. $3a(-4a - 6)$
   $-12a^2 - 18a$
5. $6b(3b^2 + 2b - 4)$
   $18b^3 + 12b^2 - 24b$
6. $3c^2(2c^2 - 4c + 3)$
   $6c^4 - 12c^3 + 9c^2$
7. $-2d(4d^2 + 3d - 2)$
   $-8d^3 - 6d^2 + 4d$
8. $5e^2(-3e^2 - 2e - 3)$
   $-15e^4 - 10e^3 - 15e^2$
9. $4f(-3f^3 + 2f^2 + 6)$
   $-12f^4 + 8f^3 + 24f$

8-2 Reteaching (continued)

Multiplying and Factoring

To factor a polynomial, find the greatest common factor (GCF) of the coefficients and constants and also the GCF of the variables.

Problem

What is the factored form of $8x^4 + 12x^2 - 16x$?

Solve Find the GCF of the coefficients. Use prime factorization.

$8 = \mathbf{2} \cdot \mathbf{2} \cdot 2$

$12 = \mathbf{2} \cdot \mathbf{2} \cdot 3$

$16 = \mathbf{2} \cdot \mathbf{2} \cdot 2 \cdot 2$

The GCF of the numbers is 4.

Each term has a variable. Remember, $x = x^1$.
The GCF is the least exponent.
The GCF of the variables is x.
The GCF is $4x$. Combine the GCFs.

Factor out the GCF of each term.

$4(2 + 3 - 4)$ Factor the coefficients.

$4x(2x^3 + 3x - 4)$ Insert the variables.

Check $4x(2x^3 + 3x - 4) = 8x^4 + 12x^2 - 16x$ Check by multiplying.

Solution: The factored form of $8x^4 + 12x^2 - 16x$ is $4x(2x^3 + 3x - 4)$.

Exercises

Find the GCF of the terms of each polynomial.

10. $12x^2 - 6x$
$6x$

11. $4y^2 + 12y + 8$
4

12. $6z^3 + 15z^2 - 9z$
$3z$

Factor each polynomial.

13. $8a + 10$
$2(4a + 5)$

14. $12b^2 - 18b$
$6b(2b - 3)$

15. $9c^3 + 12c^2$
$3c^2(3c + 4)$

16. $5d^3 - 10d^2 + 20d$
$5d(d^2 - 2d + 4)$

17. $6e^2 + 10e - 8$
$2(3e^2 + 5e - 4)$

18. $8g^3 - 24g^2 + 16g$
$8g(g^2 - 3g + 2)$

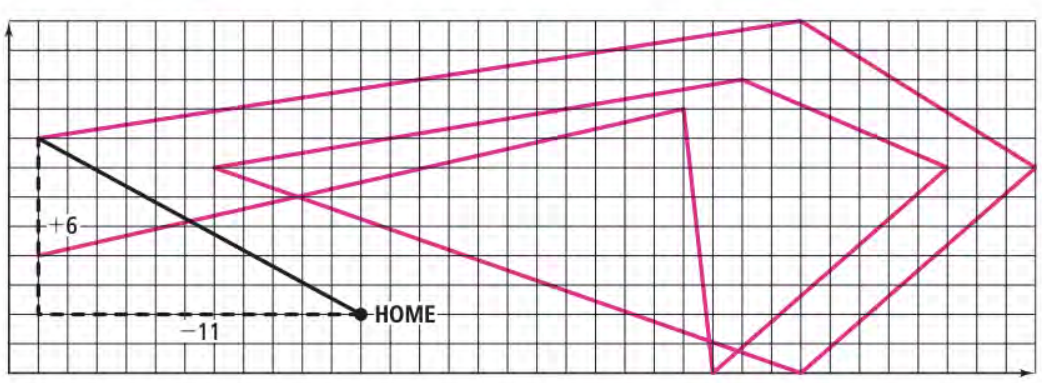
8-2 Puzzle: Getting Back Home

Multiplying and Factoring

- Complete the factorization of each expression below. The first one has been done for you.
- Use the factor *in parentheses* and the directions at the right to first move *right* or *left*, then *up* or *down* on the grid. For $3x^2(-11x + 6)$, you go *left* 11 units (-11) and then *up* 6 units $(+6)$ according to the directions at the right.
- Connect each segment to the next segment as you complete the problems.

	Horizontal move/Vertical move
1. $-33x^3 + 18x^2 = \underline{3x^2(-11x + 6)}$	coefficient of x, then constant term
2. $-26x^4 - 4x^3 - 5x^2 = -x^2(\underline{26x^2 + 4x + 5})$	coefficient of x^2, then coefficient of x
3. $31x^4 + 8x^3 - 5x^2 = x^2(\underline{31x^2 + 8x - 5})$	coefficient of x, then constant term
4. $-40x^6 - 35x^4 + 15x^3 = 5x^3(\underline{-8x^3 - 7x + 3})$	coefficient of x^3, then coefficient of x
5. $15x^5 - 100x^3 + 35x^2 = 5x^2(\underline{3x^3 - 20x + 7})$	coefficient of x, then constant term
6. $36x^7 - 4x^6 + 6x^5 = 2x^5(\underline{18x^2 - 2x + 3})$	coefficient of x^2, then constant term
7. $-28x^6 + 12x^5 - 12x^2 = -4x^2(\underline{7x^4 - 3x^3 + 3})$	coefficient of x^4, then coefficient of x^3
8. $15x^6 - 24x^5 - 21x^2 = 3x^2(\underline{5x^4 - 8x^3 - 7})$	coefficient of x^3, then constant term
9. $22x^9 - 2x^4 + 18x^3 = 2x^2(\underline{11x^7 - x^2 + 9x})$	coefficient of x^2, then coefficient of x
10. $11x^9 - 22x^8 - 5x^5 = x^2(\underline{11x^7 - 22x^6 - 5x^3})$	coefficient of x^6, then coefficient of x^3

To get HOME from your end point, move __11__ units left/right (circle one) and __2__ units up/down (circle one).



8-2 Think About a Plan

Multiplying and Factoring

a. Factor $n^2 + n$.

b. Writing Suppose n is an integer. Is $n^2 + n$ *always*, *sometimes*, or *never* an even integer? Justify your answer.

1. Factor out n from the expression.

$$n\left(\boxed{n} + \boxed{1}\right)$$

2. What are the two factors? n, $n + 1$

3. What is an integer? positive and negative whole numbers and zero

4. Are n and $n + 1$ consecutive integers? Explain. yes; the next number after n is $n + 1$

5. What do you know about the product of odd and even integers?

EVEN × EVEN = even

ODD × ODD = odd

EVEN × ODD = even

ODD × EVEN = even

6. Out of two consecutive integers, how many are odd? 1

7. Is the product of consecutive integers *odd* or *even*? Explain. even; Two consecutive integers must be an odd integer and an even integer. If 1 factor is even, the product will be even.

8. $n^2 + n$ is always an even integer because it is the product of two consecutive integers.

8-2 Practice

Form G

Multiplying and Factoring

Simplify each product.

1. $2x(x + 8)$
   $2x^2 + 16x$
2. $(n + 7)5n$
   $5n^2 + 35n$
3. $6h^2(7 + h)$
   $6h^3 + 42h^2$
4. $-b^2(b - 10)$
   $-b^3 + 10b^2$
5. $-3c(8 + 2c - c^3)$
   $3c^4 - 6c^2 - 24c$
6. $y(2y^2 - 3y + 6)$
   $2y^3 - 3y^2 + 6y$
7. $4t(t^2 - 6t + 2)$
   $4t^3 - 24t^2 + 8t$
8. $-m(4m^3 - 8m^2 + m)$
   $-4m^4 + 8m^3 - m^2$
9. $7j(-2j^2 - 8j - 3)$
   $-14j^3 - 56j^2 - 21j$
10. $-t^2(2t^4 + 4t - 8)$
    $-2t^6 - 4t^3 + 8t^2$
11. $2k(-3k^3 + k^2 - 10)$
    $-6k^4 + 2k^3 - 20k$
12. $8a^2(-a^7 + 7a - 7)$
    $-8a^9 + 56a^3 - 56a^2$
13. $4v^3(2v^2 - 3v + 5)$
    $8v^5 - 12v^4 + 20v^3$
14. $5d(-d^3 + 2d^2 - 3d)$
    $-5d^4 + 10d^3 - 15d^2$
15. $11w(w^2 + 2w + 6)$
    $11w^3 + 22w^2 + 66w$

Find the GCF of the terms of each polynomial.

16. $15x + 27$
    3
17. $6w^3 - 14w$
    $2w$
18. $63s + 45$
    9
19. $72y^5 + 18y^2$
    $18y^2$
20. $-18q^3 - 6q^2$
    $-6q^2$
21. $108f^3 - 54$
    54
22. $b^3 + 5b^2 - 20b$
    b
23. $9m^3 + 30m - 24$
    3
24. $4p^3 + 12p^2 - 18p$
    $2p$
25. $2e^2 + 12e - 22$
    2
26. $14b^3 + 21b^2 - 42b$
    $7b$
27. $-12x^3 + 24x^2 - 16x$
    $4x$
28. $8a^4 + 24a^3 - 40a^2$
    $8a^2$
29. $36j^3 - 3j^2 - 15j$
    $3j$
30. $12j^8 + 30j^4 - 6j^3$
    $6j^3$

Factor each polynomial.

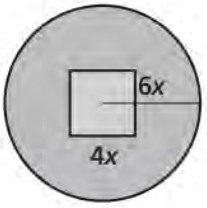
31. $12x - 9$
    $3(4x - 3)$
32. $18s^2 + 54$
    $18(s^2 + 3)$
33. $108t^2 - 60t$
    $12t(9t - 5)$
34. $-20w^2 + 16w$
    $-4w(5w - 4)$
35. $32y^3 + 8y^2$
    $8y^2(4y + 1)$
36. $300d^2 - 175d$
    $25d(12d - 7)$
37. $12n^3 - 36n^2 + 18$
    $6(2n^3 - 6n^2 + 3)$
38. $40t^3 + 25t^2 + 80t$
    $5t(8t^2 + 5t + 16)$
39. $42x^4 - 56x^3 + 28x^2$
    $14x^2(3x^2 - 4x + 2)$
40. $15c^4 + 24c^3 - 6c^2 + 12c$
    $3c(5c^3 + 8c^2 - 2c + 4)$
41. $8m^3 + 14m^2 + 6m$
    $2m(4m^2 + 7m + 3)$
42. $10x^2 + 50x - 25$
    $5(2x^2 + 10x - 5)$
43. $36p^4 + 14p^3 + 35p^2$
    $p^2(36p^2 + 14p + 35)$
44. $9a^5 + 27a^4 + 63a^2$
    $9a^2(a^3 + 3a^2 + 7)$
45. $4b^4 + 20b^3 + 12b$
    $4b(b^3 + 5b^2 + 3)$
46. $x^6 - x^4 + x^2$
    $x^2(x^4 - x^2 + 1)$
47. $34g^3 + 51g^2 + 17g$
    $17g(2g^2 + 3g + 1)$
48. $18h^4 - 27h^2 + 18h$
    $9h(2h^3 - 3h + 2)$

8-2 Practice (continued)

Form G

Multiplying and Factoring

49. A circular hedge surrounds a sculpture on a square base. The radius of the hedge is $6x$. The side length of the square sculpture base is $4x$. What is the area of the hedge? Write your answer in factored form. $4x^2(9\pi - 4)$



50. Suppose you are making a giant chocolate chip cookie for a raffle. You roll out a square slab of cookie dough. Then you use a circular plate that touches the edges of the square slab of cookie dough and cut the cookie out of the dough. What is the area of the extra dough? Write your answer in factored form. $r^2(4 - \pi)$

Simplify. Write in standard form.

51. $-3x(4x^2 - 6x + 12)$
$-12x^3 + 18x^2 - 36x$

52. $-7y^2(-4y^3 + 6y)$
$28y^5 - 42y^3$

53. $9a(-3a^2 + a - 5)$
$-27a^3 + 9a^2 - 45a$

54. $p(p + 4) - 2p(p - 8)$
$-p^2 + 20p$

55. $t(t + 4) - t(4t^2 - 2)$
$-4t^3 + t^2 + 6t$

56. $6c(2c^2 - 4) - c(8c)$
$12c^3 - 8c^2 - 24c$

57. $-5m(2m^3 - 7m^2 + m)$
$-10m^4 + 35m^3 - 5m^2$

58. $2q(q + 1) - q(q - 1)$
$q^2 + 3q$

59. $-n^2(-6n^2 + 2n)$
$6n^4 - 2n^3$

Factor each polynomial.

60. $15xy^4 + 60x^2y^3$
$15xy^3(y + 4x)$

61. $8m^3n^4 + 32mn^2$
$8mn^2(m^2n^2 + 4)$

62. $26a^5b^2 + 51a^4$
$a^4(26ab^2 + 51)$

63. $36j^2k^4 + 24j^4k^2$
$12j^2k^2(3k^2 + 2j^2)$

64. $12w^4x^3 - 42wx^2$
$6wx^2(2w^3x - 7)$

65. $54c^2d^3 - 36c^3d^2$
$18c^2d^2(3d - 2c)$

66. $12st^4 + 46s^3t^4$
$2st^4(6 + 23s^2)$

67. $9v^6w^3 + 33v^4w^5$
$3v^4w^3(3v^2 + 11w^2)$

68. $11e^3f^3 + 132e^2f^4$
$11e^2f^3(e + 12f)$

69. Error Analysis A student factored the polynomial at the right. Describe and correct the error made in factoring.
The student did not find the correct GCF. $7x^2(9x^2 - 2x + 5)$

$$\cancel{63x^4 - 14x^3 + 35x^2 = 7x(9x^3 - 2x^2 + 5x)}$$

70. Reasoning The GCF of two numbers j and k is 8. What is the GCF of $2j$ and $2k$? Justify your answer. 16 The GCF will be the product of 2 and 8.

71. A cylinder has a radius of $3m^2n$ and a height of $7mn$. The formula for the volume of a cylinder is $V = \pi r^2h$, where r is the radius and h is the height. What is the volume of the cylinder? Simplify your answer. $63\pi m^5n^3$

8-2 Practice

Form K

Multiplying and Factoring

Simplify each product.

1. $3w(w + 2)$
   $3w^2 + 6w$

2. $(z + 5)2z$
   $2z^2 + 10z$

3. $3m^2(4 + m)$
   $12m^2 + 3m^3$

4. $2p(p^2 - 6p + 1)$
   $2p^3 - 12p^2 + 2p$

5. $-y(5y^3 - 3y^2 + 2y)$
   $-5y^4 + 3y^3 - 2y^2$

6. $3a(-3a^2 + 2a - 7)$
   $-9a^3 + 6a^2 - 21a$

7. $6x^3(3x^2 - x + 10)$
   $18x^5 - 6x^4 + 60x^3$

8. $-4h(-h^3 - 8h^2 + 2h)$
   $4h^4 + 32h^3 - 8h^2$

9. $4n(n^2 + 5n + 6)$
   $4n^3 + 20n^2 + 24n$

Find the GCF of the terms of each polynomial.

10. $16q + 32$
    16

11. $4t^3 - 24t$
    $4t$

12. $32y - 24$
    8

13. $x^3 + 3x^2 + 5x$
    x

14. $5d^3 + 20d - 35$
    5

15. $2m^3 + 10m^2 + 12m$
    $2m$

16. $7g^4 + 21g^3 - 14g^2$
    $7g^2$

17. $15z^3 + 3z^2 - 27z$
    $3z$

18. $33w^7 + 55w^5 - 22w^3$
    $11w^3$

Factor each polynomial.

19. $9t - 3$
    $3(3t - 1)$

20. $12j^3 + 28$
    $4(3j^3 + 7)$

21. $72x^2 - 63x$
    $9x(8x - 7)$

22. $12k^3 - 9k^2 + 6$
    $3(4k^3 - 3k^2 + 2)$

23. $30n^3 + 18n^2 + 54n$
    $6n(5n^2 + 3n + 9)$

24. $32z^4 - 80z^3 + 112z^2$
    $16z^2(2z^2 - 5z + 7)$

25. $12n^4 + 16n^3 + 20n^2$
    $4n^2(3n^2 + 4n + 5)$

26. $24y^6 + 36y^4 + 42y^2$
    $6y^2(4y^4 + 6y^2 + 7)$

27. $7q^5 + 21q^3 - 49q$
    $7q(q^4 + 3q^2 - 7)$

8-2 Practice (continued)

Form K

Multiplying and Factoring

28. You are painting a rectangular wall with length $5x^2$ ft and width $12x$ ft. There is a rectangular door that measures x ft by $2x$ ft that will not be painted. What is the area of the wall that is to be painted? Write your answer in factored form.
$2x^2(30x - 1)$

Simplify. Write in standard form.

29. $-3m(2m^2 - 5m + 10)$
$-6m^3 + 15m^2 - 30m$

30. $-5t^2(-6t^3 + 12t)$
$30t^5 - 60t^3$

31. $10x(-4x^2 + x - 3)$
$-40x^3 + 10x^2 - 30x$

32. $-2v(3v^3 - 6v^2 + 2v)$
$-6v^4 + 12v^3 - 4v^2$

33. $5y(y + 2) - y(y - 3)$
$4y^2 + 13y$

34. $-2b^2(-4b^2 + 3b)$
$8b^4 - 6b^3$

Factor each polynomial.

35. $13cd^3 + 39c^2d^2$
$13cd^2(d + 3c)$

36. $5x^3y^4 - 25xy^2$
$5xy^2(x^2y^2 - 5)$

37. $42m^5n + 28m^4$
$14m^4(3mn + 2)$

38. $36fg^2 + 54f^2g^4$
$18fg^2(2 + 3fg^2)$

39. $8s^8t^4 + 20s^4t^3$
$4s^4t^3(2s^4t + 5)$

40. $12a^2b^5 + 156a^2b^3$
$12a^2b^3(b^2 + 13)$

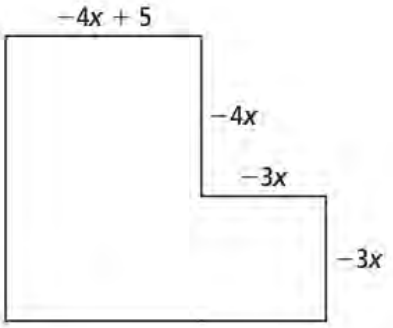
41. Open-Ended Write a quadratic monomial and a cubic trinomial. Then find their product and write it in standard form.
Answers may vary. Sample: x^2 and $2x^3 + x^2 + x$; $2x^5 + x^4 + x^3$

42. A rectangle has a length of $6x^3y^2 - 1$ and a width of $3xy + 2$. The formula for the perimeter of a rectangle is $P = 2l + 2w$, where l is the length and w is the width. What is the perimeter of the rectangle? Simplify your answer. $12x^3y^2 + 6xy + 2$

8-2 Enrichment

Multiplying and Factoring

To find the area of irregular figures, split the figure into simple figures and then add the areas of each figure.



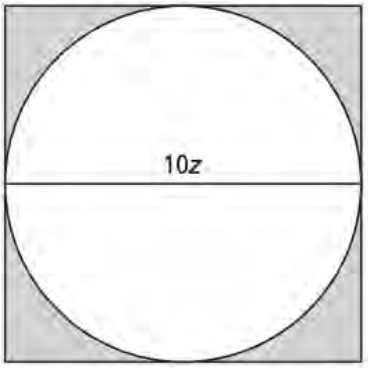
1. What is the area of the figure to the right?

   $37x^2 - 35x$

2. What is the perimeter of the figure?

   $-28x + 10$

A circle is inscribed in a square as shown.



3. What is the area of the circle?

   $25\pi z^2$

4. What is the area of the square?

   $100z^2$

5. What is the area of the shaded region?

   $100z^2 - 25\pi z^2$, or $25z^2(4 - \pi)$

6. The area of a right triangle is $10y^3 + 5y^2 + 37.5y$. The length of base of the triangle is a monomial with a whole number coefficient. The length of the height is a trinomial. Factor the polynomial to find the base and height of the triangle. (Remember to multiply the area by 2 first because $A_{\text{triangle}} = \frac{b \times h}{2}$.)

   Base = $5y$

   Height = $4y^2 + 2y + 15$

8-3 Additional Vocabulary Support

Multiplying Binomials

Use the Distributive Property to find the simplified form of $(3x + 2)(4x - 3)$.

$(3x + 2)(4x - 3)$	Write the problem.
$3x(4x - 3) + 2(4x - 3)$	Distribute the second factor, $4x - 3$.
$12x^2 - 9x + 2(4x - 3)$	Distribute 3x.
$12x^2 - 9x + 8x - 6$	Distribute 2.
$12x^2 - x - 6$	Combine like terms.

Exercises

Use the Distributive Property to find the simplified form of $(5x + 6)(2x - 4)$.

$(5x + 6)(2x - 4)$	Write the problem.
$5x(2x - 4) + 6(2x - 4)$	Distribute the second factor, $2x - 4$.
$10x^2 - 20x + 6(2x - 4)$	Distribute $5x$.
$10x^2 - 20x + 12x - 24$	Distribute 6.
$10x^2 - 8x - 24$	Combine like terms.

Use the Distributive Property to find the simplified form of $(7x - 3)(4x + 6)$.

$(7x - 3)(4x + 6)$	Write the problem.
$7x(4x + 6) - 3(4x + 6)$	Distribute the second factor, $4x + 6$.
$28x^2 + 42x - 3(4x + 6)$	Distribute $7x$.
$28x^2 + 42x - 12x - 18$	Distribute -3.
$28x^2 + 30x - 18$	Combine like terms.

8-3 Reteaching

Multiplying Binomials

You can multiply binomials by using the FOIL method. FOIL stands for First, Outer, Inner, and Last.

Problem

What is the simplified form of $(4x + 3)(2x + 6)$?

Use the FOIL method to simplify the binomial.

Solve

$4x \cdot 2x = 8x^2$	Multiply the First terms.
$4x \cdot 6 = 24x$	Multiply the Outer terms.
$3 \cdot 2x = 6x$	Multiply the Inner terms.
$3 \cdot 6 = 18$	Multiply the Last terms.
$8x^2 + 24x + 6x + 18$	Add the products.
$8x^2 + 30x + 18$	Add the like terms.

Check Substitute any number for x. Try $x = 2$. If the two sides of the equation are equal the simplification may be correct.

$$(4x + 3)(2x + 6) \stackrel{?}{=} 8x^2 + 30x + 18$$

$$(4 \cdot 2 + 3)(2 \cdot 2 + 6) \stackrel{?}{=} (8 \cdot 2^2) + (30 \cdot 2) + 18$$

$$(11)(10) \stackrel{?}{=} 32 + 60 + 18$$

$$110 = 110 \checkmark$$

Solution: The simplified form of $(4x + 3)(2x + 6)$ is $8x^2 + 30x + 18$.

Exercises

Simplify each product.

1. $(a + 6)(a - 3)$
   $a^2 + 3a - 18$
2. $(b - 4)(b + 5)$
   $b^2 + b - 20$
3. $(c + 3)(c + 7)$
   $c^2 + 10c + 21$
4. $(2d + 4)(3d - 2)$
   $6d^2 + 8d - 8$
5. $(4e - 5)(3e + 3)$
   $12e^2 - 3e - 15$
6. $(3f - 2)(2f - 4)$
   $6f^2 - 16f + 8$
7. $(5g + 3)(g - 3)$
   $5g^2 - 12g - 9$
8. $(4h + 4)(2h + 5)$
   $8h^2 + 28h + 20$
9. $(3j - 5)(4j - 3)$
   $12j^2 - 29j + 15$

8-3 Reteaching (continued)

Multiplying Binomials

To multiply a trinomial by a binomial, use the same steps as you would to multiply a 3-digit number by a 2-digit number. Find the partial products for each term of the binomial and then add the like terms of the partial products.

Problem

What is the simplified form of $(2x^2 + 3x - 4)(3x + 2)$?

Solve Start by arranging the polynomials vertically.

Multiply each part of the trinomial by 2.

$$\begin{array}{r} \mathbf{2x^2 + 3x - 4} \\ \underline{3x + \mathbf{2}} \\ \mathbf{4x^2 + 6x - 8} \end{array} \qquad \begin{array}{r} 2x^2 \cdot 2 = 4x^2 \\ 3x \cdot 2 = 6x \\ -4 \cdot 2 = -8 \end{array}$$

Multiply each part of the trinomial by $3x$.

$$\begin{array}{r} \mathbf{2x^2 + 3x - 4} \\ \underline{\mathbf{3x} + 2} \\ 4x^2 + 6x - 8 \\ \mathbf{6x^3 + 9x^2 - 12x} \end{array} \qquad \begin{array}{r} 2x^2 \cdot 3x = 6x^3 \\ 3x \cdot 3x = 9x^2 \\ -4 \cdot 3x = -12x \end{array}$$

Add the partial products.

$$\begin{array}{r} 4x^2 + 6x - 8 \\ \underline{6x^3 + 9x^2 - 12x \qquad} \\ 6x^3 + 13x^2 - 6x - 8 \end{array}$$

Check Substitute any number for x. Try $x = 2$. If the two sides of the equation are equal, the simplification may be correct.

$$(2x^2 + 3x - 4)(3x + 2) \stackrel{?}{=} 6x^3 + 13x^2 - 6x - 8$$
$$(8 + 6 - 4)(6 + 2) \stackrel{?}{=} 48 + 52 - 12 - 8$$
$$80 = 80 \checkmark$$

Solution: The simplified form of $(2x^2 + 3x - 4)(3x + 2)$ is $6x^3 + 13x^2 - 6x - 8$.

Exercises

Simplify each product.

10. $(w^2 + 3w - 4)(2w + 3)$
$2w^3 + 9w^2 + w - 12$

11. $(x^2 - 8x + 6)(3x - 4)$
$3x^3 - 28x^2 + 50x - 24$

12. $(2y^2 + 4y - 5)(4y + 2)$
$8y^3 + 20y^2 - 12y - 10$

13. $(3z^2 - 6z + 4)(4z + 1)$
$12z^3 - 21z^2 + 10z + 4$

8-3 Puzzle: The Binomial Code

Multiplying Binomials

Multiply the binomials in the tables, then complete the columns to the right. The first one has been done for you. Translate the numbers in the *shaded* boxes to letters using the code below. Then unscramble the letters to complete the phrase at the bottom of the page.

A → 1 B → 2 C → 3 · · · X → 24 Y → 25 Z → 26

First Word		x^2	x	Constant	Letter
$(x + 1)(x + 5)$	$x^2 + 6x + 5$	1	6	5	O
$(x + 4)(x + 5)$	$x^2 + 9x + 20$	1	9	20	T
$(x + 2)(x + 7)$	$x^2 + 9x + 14$	1	9	14	N
$(x + 9)(x + 1)$	$x^2 + 10x + 9$	1	10	9	I

Second Word		x^2	x	Constant	Letter
$(9x - 1)(x + 5)$	$9x^2 + 44x - 5$	9	44	−5	I
$(x - 5)(x - 1)$	$x^2 - 6x + 5$	1	−6	5	E
$(x + 1)(x + 1)$	$x^2 + 2x + 1$	1	2	1	K

Third Word		x^2	x	Constant	Letter
$(x + 7)(x + 2)$	$x^2 + 9x + 14$	1	9	14	S
$(x + 3)(x + 3)$	$x^2 + 6x + 9$	1	6	9	S
$(3x - 2)(x - 5)$	$3x^2 - 17x + 10$	3	−17	10	C
$(x + 3)(x + 2)$	$x^2 + 5x + 6$	1	5	6	E
$(5x + 1)(7x + 3)$	$35x^2 + 22x + 3$	35	22	3	C

Fourth Word		x^2	x	Constant	Letter
$(x + 2)(x + 2)$	$x^2 + 4x + 4$	1	4	4	N
$(x + 4)(x + 5)$	$x^2 + 9x + 20$	1	9	20	I

Fifth Word		x^2	x	Constant	Letter
$(x + 2)(x + 1)$	$x^2 + 3x + 2$	1	3	2	M
$(5x + 2)(4x + 1)$	$20x^2 + 13x + 2$	20	13	2	T
$(x + 11)(x - 11)$	$x^2 - 121$	1	0	−121	A

MESSAGE: N O T H I N G L I K E S U C C E S S I N M A T H!

8-3 Think About a Plan

Multiplying Binomials

Geometry The dimensions of a rectangular prism are n, $n + 7$, and $n + 8$. Use the formula $V = lwh$ to write a polynomial in standard form for the volume of the prism.

Know

1. What are the dimensions of the rectangular prism? n, $n + 7$, $n + 8$

2. What is the formula for the volume of a rectangular prism? $V = lwh$

3. In the volume formula, what do l, w, and h represent? length, width, height

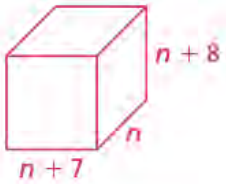
4. Explain how to write a polynomial in standard form. The terms are arranged in order of degree, with the highest degree first.

Need

5. To solve the problem you need to find a polynomial in standard form that represents the volume of the rectangular prism.

Plan

6. Draw a diagram of the rectangular prism and label the information you know.



7. Write an expression for the volume of the rectangular prism.
$V = n(n + 7)(n + 8)$

8. Write the volume of the rectangular prism as a polynomial in standard form.
$n^3 + 15n^2 + 56n$

8-3 Practice

Form G

Multiplying Binomials

Simplify each product using the Distributive Property.

1. $(x + 3)(x + 8)$
$x^2 + 11x + 24$

2. $(y - 4)(y + 7)$
$y^2 + 3y - 28$

3. $(m + 9)(m - 3)$
$m^2 + 6m - 27$

4. $(c - 6)(c - 4)$
$c^2 - 10c + 24$

5. $(2r - 5)(r + 3)$
$2r^2 + r - 15$

6. $(3x + 1)(5x - 3)$
$15x^2 - 4x - 3$

7. $(d + 2)(4d - 3)$
$4d^2 + 5d - 6$

8. $(5t - 1)(3t - 2)$
$15t^2 - 13t + 2$

9. $(a + 11)(11a + 1)$
$11a^2 + 122a + 11$

Simplify each product using a table.

10. $(x + 3)(x - 5)$
$x^2 - 2x - 15$

11. $(a - 2)(a - 13)$
$a^2 - 15a + 26$

12. $(w - 4)(w + 8)$
$w^2 + 4w - 32$

13. $(5h - 3)(h + 7)$
$5h^2 + 32h - 21$

14. $(x - 3)(2x + 3)$
$2x^2 - 3x - 9$

15. $(2p + 1)(6p + 4)$
$12p^2 + 14p + 4$

Simplify each product using the FOIL method.

16. $(2x - 6)(x + 3)$
$2x^2 - 18$

17. $(n - 5)(3n - 4)$
$3n^2 - 19n + 20$

18. $(4p^2 + 2)(3p - 1)$
$12p^3 - 4p^2 + 6p - 2$

19. $(a + 7)(a - 3)$
$a^2 + 4a - 21$

20. $(x + 3)(3x - 2)$
$3x^2 + 7x - 6$

21. $(k - 9)(k + 5)$
$k^2 - 4k - 45$

22. $(b - 5)(b - 11)$
$b^2 - 16b + 55$

23. $(4m - 1)(m + 4)$
$4m^2 + 15m - 4$

24. $(7z + 3)(4z - 6)$
$28z^2 - 30z - 18$

25. $(2h + 6)(5h - 3)$
$10h^2 + 24h - 18$

26. $(3w + 12)(w + 3)$
$3w^2 + 21w + 36$

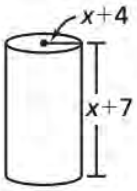
27. $(6c - 2)(9c - 8)$
$54c^2 - 66c + 16$

8-3 Practice (continued)

Form G

Multiplying Binomials

28. What is the surface area of the cylinder at the right? Write your answer in simplified form.



$4\pi x^2 + 38\pi x + 88\pi$

29. The radius of a cylindrical popcorn tin is $(3x + 1)$ in. The height of the tin is three times the radius. What is the surface area of the cylinder? Write your answer in simplified form.

$72\pi x^2 + 48\pi x + 8\pi$

30. The radius of a cylindrical tennis ball can is $(2x + 1)$ cm. The height of the tennis ball can is six times the radius. What is the surface area of the cylinder? Write your answer in simplified form.

$56\pi x^2 + 56\pi x + 14\pi$

Simplify each product.

31. $(x + 3)(x^2 - 2x + 4)$

$x^3 + x^2 - 2x + 12$

32. $(k^2 - 5k + 2)(k - 5)$

$k^3 - 10k^2 + 27k - 10$

33. $(3a^2 + a + 4)(2a - 6)$

$6a^3 - 16a^2 + 2a - 24$

34. $(2x^2 + 2x - 6)(3x - 4)$

$6x^3 - 2x^2 - 26x + 24$

35. $(4g + 5)(2g^2 - 7g + 3)$

$8g^3 - 18g^2 - 23g + 15$

36. $(m^2 - 2m + 7)(3m + 6)$

$3m^3 + 9m + 42$

37. $(2c + 8)(2c^2 - 4c - 1)$

$4c^3 + 8c^2 - 34c - 8$

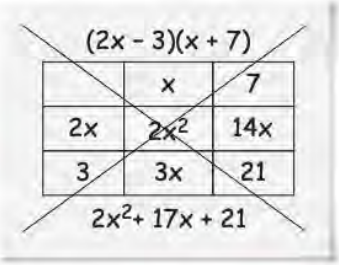
38. $(t + 8)(3t^2 + 4t + 5)$

$3t^3 + 28t^2 + 37t + 40$

39. A medical center's rectangular parking lot currently has a length of 30 meters and a width of 20 meters. The center plans to expand both the length and the width of the parking lot by $2x$ meters. What polynomial in standard form represents the area of the expanded parking lot?

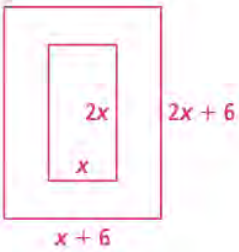
$4x^2 + 100x + 600$

40. Error Analysis Describe and correct the error made in finding the product.



In the table, the 3 should be -3. Therefore, $3x$ should be $-3x$ and 21 should be -21. The answer is $2x^2 + 11x - 21$.

41. Multi Step The height of a painting is twice its width x. You want a 3 inch wide wooden frame for the painting. The area of the frame alone is 216 square inches.



a. Draw a diagram that represents this situation.

b. Write a variable expression for the area of the frame alone.
$18x + 36$

c. What are the dimensions of the frame? length is 26; width is 16

8-3 Practice

Form K

Multiplying Binomials

Simplify each product using the Distributive Property.

1. $(b - 2)(b + 1)$ $b^2 - b - 2$

2. $(x + 6)(x + 5)$ $x^2 + 11x + 30$

3. $(3n + 1)(n - 8)$ $3n^2 - 23n - 8$

4. $(2t - 7)(t - 5)$ $2t^2 - 17t + 35$

5. $(y + 3)(y + 7)$ $y^2 + 10y + 21$

6. $(b - 6)(b + 3)$ $b^2 - 3b - 18$

Simplify each product using a table.

7. $(x + 1)(x - 11)$
$x^2 - 10x - 11$

8. $(h - 2)(3h + 5)$
$3h^2 - h - 10$

9. $(8w - 3)(4w - 7)$
$32w^2 - 68w + 21$

10. $(3c + 13)(13c + 3)$
$39c^2 + 178c + 39$

11. $(3a + 2)(a - 2)$
$3a^2 - 4a - 4$

12. $(t + 7)(2t - 4)$
$2t^2 + 10t - 28$

13. $(3q^2 + 6)(2q - 5)$
$6q^3 - 15q^2 + 12q - 30$

14. $(x + 6)(x - 7)$
$x^2 - x - 42$

15. $(p - 10)(2p + 5)$
$2p^2 - 15p - 50$

16. $(j - 12)(j - 11)$
$j^2 - 23j + 132$

17. $(3z - 4)(7z - 5)$
$21z^2 - 43z + 20$

18. $(2m + 11)(6m - 1)$
$12m^2 + 64m - 11$

19. $(7h + 6)(7h - 6)$
$49h^2 - 36$

20. $(-3z + 7)(4z - 8)$
$-12z^2 + 52z - 56$

21. $(-3t + 5)(-3t - 2)$
$9t^2 - 9t - 10$

8-3 Practice (continued)

Form K

Multiplying Binomials

22. The radius of a circle is $(7x + 3)$ cm. Write an expression to represent the area of the circle in simplified form. $49\pi x^2 + 42\pi x + 9\pi$ cm^2

23. A rectangle has a length of $(x + 2)$ in. and a width of $(2x + 3)$ in. Find an expression that represents the area of the rectangle. Write the expression in simplified form. $2x^2 + 7x + 6$ in.^{2}

Simplify each product using the FOIL method.

24. $(x + 4)(x + 6)$
$x^2 + 10x + 24$

25. $(a - 5)(2a - 6)$
$2a^2 - 16a + 30$

26. $(6d^2 + 4)(8d - 3)$
$48d^3 - 18d^2 + 32d - 12$

27. $(t - 4)(t - 9)$
$t^2 - 13t + 36$

28. $(n + 8)(2n - 7)$
$2n^2 + 9n - 56$

29. $(f - 7)(f + 3)$
$f^2 - 4f - 21$

Simplify each product.

30. $(c + 4)(c^2 - 3c + 5)$
$c^3 + c^2 - 7c + 20$

31. $(p^2 - 2p + 5)(p - 7)$
$p^3 - 9p^2 + 19p - 35$

32. $(4x^2 + 2x + 3)(3x - 8)$
$12x^3 - 26x^2 - 7x - 24$

33. $(5t^2 + 3t - 11)(6t - 1)$
$30t^3 + 13t^2 - 69t + 11$

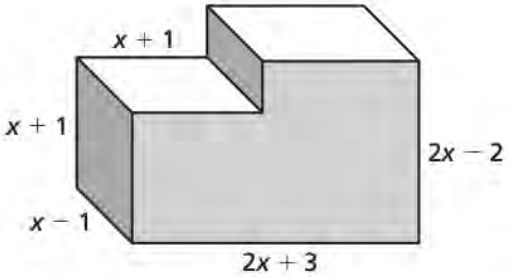
34. A community center is expanding the size of its rectangular meeting hall. The hall is currently 300 ft long and 150 ft wide. The center plans to expand both the length and the width of the meeting hall by $3x$ ft. What polynomial in standard form represents the area of the expanded meeting hall?
$9x^2 + 1350x + 45{,}000$ ft^2

35. Open-Ended Write a cubic monomial and a fourth-degree trinomial. Then find their product and write it in standard form.
Answers may vary. Sample: $2x^3$ and $x^4 + 2x + 3$; $2x^7 + 4x^4 + 6x^3$

8-3 Enrichment

Multiplying Binomials

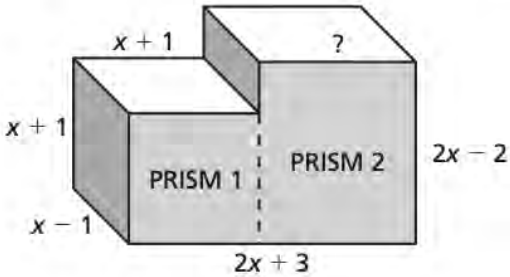
You can find the volume of irregular figures by dividing the figure into smaller rectangular prisms, finding the volume of each separate figure, and then adding them together. The figure to the right can be divided into two rectangular prisms.



$$V_1 = (x + 1)(x + 1)(x - 1)$$
$$= (x^2 + 2x + 1)(x - 1)$$
$$= x^3 + x^2 - x - 1$$

Subtract to find the length of Prism 2.

$$(2x + 3) - (x + 1) = x + 2$$



$$V_2 = (x + 2)(x - 1)(2x - 2)$$
$$= (x^2 + x - 2)(2x - 2)$$
$$= 2x^3 - 6x + 4$$

$$V_{\text{Total}} = (x^3 + x^2 - x - 1) + (2x^3 - 6x + 4)$$
$$= 3x^3 + x^2 - 7x + 3$$

You can also find the volume of an irregular figure by finding the volume of the whole figure, as if no pieces were cut away. Next, find the volume of the cut away piece, and then subtract that volume from the whole. Prism 2 is $x - 3$ taller than Prism 1.

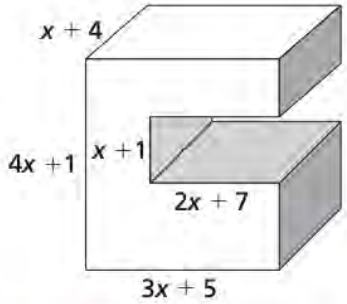
$$V_{\text{Whole}} = (x - 1)(2x + 3)(2x - 2) = (2x^2 + x - 3)(2x - 2) = 4x^3 - 2x^2 - 8x + 6$$

$$V_{\text{Piece}} = (x - 1)(x + 1)(x - 3) = (x^2 - 1)(x - 3) = x^3 - 3x^2 - x + 3$$

$$V_{\text{Total}} = (4x^3 - 2x^2 - 8x + 6) - (x^3 - 3x^2 - x + 3) = 3x^3 + x^2 - 7x + 3$$

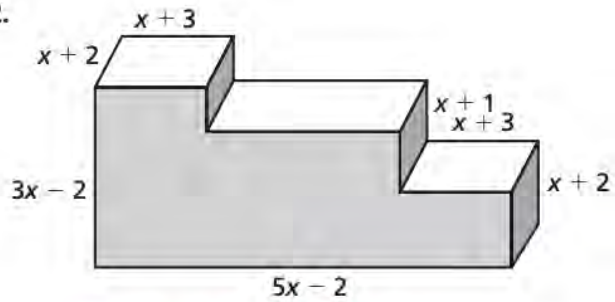
What is the volume of each figure? Write your answer as a polynomial in standard form.

1.



$10x^3 + 54x^2 + 54x - 8$

2.



$10x^3 + 25x^2 - 14x - 48$

8-4 Additional Vocabulary Support

Multiplying Special Cases

Use the list below to complete the diagram.

The square of a binomial is the square of the first term plus twice the product of the two terms plus the square of the last term.	$(x + 3)(x - 3) =$ $x^2 - 3^2 = x^2 - 9$	The product of the sum and difference of the same two terms is the difference of their squares.
$(a + b)^2 = a^2 + 2ab + b^2$	$(a - b)^2 = a^2 - 2ab + b^2$	$(a + b)(a - b) = a^2 - b^2$

The Square of a Binomial

The square of the binomial is the square of the first term plus twice the product of the two terms plus the square of the last term.

$(a + b)^2 =$
$a^2 + 2ab + b^2$

$(a - b)^2 =$
$a^2 - 2ab + b^2$

The Product of a Sum and Difference

The product of the sum and difference of the same two terms is the difference of their squares.

$(a + b)(a - b) =$
$a^2 - b^2$

$(x + 3)(x - 3) =$
$x^2 - 3^2 = x^2 - 9$

8-4 Reteaching

Multiplying Special Cases

A binomial is squared when it is multiplied by itself. The square of a binomial is the square of the first term plus the twice the product of the two terms plus the square of the last term. This can be expressed as $(a + b)^2 = a^2 + 2ab + b^2$.

Problem

What is the simplified form of $(x + 5)^2$?

Use the rules for squaring a binomial.

Solve	$x \cdot x = x^2$	Square the first term.
	$2(5 \cdot x) = 10x$	Multiply the product of the two terms by 2.
	$5 \cdot 5 = 25$	Square the last term.

So, $(x + 5)^2 = x^2 + 10x + 25$.

Check	$(x + 5)^2 = (x + 5)(x + 5)$	Rewrite the binomials.
	$x \cdot x = x^2$	Multiply the First addends.
	$x \cdot 5 = 5x$	Multiply the Outer addends.
	$5 \cdot x = 5x$	Multiply the Inner addends.
	$5 \cdot 5 = 25$	Multiply the Last addends.
	$x^2 + 5x + 5x + 25$	Add the products.
	$x^2 + 10x + 25$	Combine the like terms.

Solution: The simplified form of $(x + 5)^2$ is $x^2 + 10x + 25$.

Exercises

Simplify each expression.

1. $(a + 7)^2$
   $a^2 + 14a + 49$
2. $(b - 4)^2$
   $b^2 - 8b + 16$
3. $(2c + 3)^2$
   $4c^2 + 12c + 9$
4. $(3d - 5)^2$
   $9d^2 - 30d + 25$
5. $(4e + 1)^2$
   $16e^2 + 8e + 1$
6. $(2f - 6)^2$
   $4f^2 - 24f + 36$
7. $(g - 10)^2$
   $g^2 - 20g + 100$
8. $(5h + 8)^2$
   $25h^2 + 80h + 64$
9. $(3j - 3)^2$
   $9j^2 - 18j + 9$
10. $(2k + 4)^2$
    $4k^2 + 16k + 16$
11. $(4m - 2)^2$
    $16m^2 - 16m + 4$
12. $(3n + 6)^2$
    $9n^2 + 36n + 36$

8-4 Reteaching (continued)

Multiplying Special Cases

The product of the sum and the difference of the same two terms produces a pattern that can be expanded algebraically as $(a + b)(a - b) = a^2 - ab + ab - b^2$. The sum of the two ab- terms is 0. Therefore, $(a + b)(a - b) = a^2 - b^2$. The product is the square of the first term minus the square of the last term.

Problem

What is the simplified form of $(2x - 3)(2x + 3)$?

Use the rules for finding the product of the sum and the difference of the same two terms.

Solve

$2x \cdot 2x = 4x^2$	Square the first term.
$3 \cdot 3 = 9$	Square the last term.

Remember, the product is the difference of the two squares.

The product is $4x^2 - 9$.

Check

Multiply the binomials using the FOIL Method.

$2x \cdot 2x = 4x^2$	Multiply the First addends.
$2x \cdot 3 = 6x$	Multiply the Outer addends.
$-3 \cdot 2x = -6x$	Multiply the Inner addends.
$-3 \cdot 3 = -9$	Multiply the Last addends.
$4x^2 + 6x - 6x - 9$	Add the products.
$4x^2 - 9$	Combine the like terms.

Solution: The simplified form of $(2x - 3)(2x + 3)$ is $4x^2 - 9$.

Exercises

Simplify each product.

13. $(p - 4)(p + 4)$
$p^2 - 16$

14. $(q + 5)(q - 5)$
$q^2 - 25$

15. $(3r + 2)(3r - 2)$
$9r^2 - 4$

16. $(4s - 6)(4s + 6)$
$16s^2 - 36$

17. $(2t - 1)(2t + 1)$
$4t^2 - 1$

18. $(5u - 3)(5u + 3)$
$25u^2 - 9$

19. $(6v - 4)(6v + 4)$
$36v^2 - 16$

20. $(3w - 8)(3w + 8)$
$9w^2 - 64$

21. $(7x - 9)(7x + 9)$
$49x^2 - 81$

8-4 Activity: Special Cases

Multiplying Special Case Binomials

This is an activity for two students. Use the following three products involving squares.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

Make ten cards by cutting out the boxes below. Turn the cards face down, and then shuffle them. Take turns choosing a card from the pile. On the blank side of the card, write a shortcut for multiplying the factors. Time each other to see who can find the fastest product using the shortcuts.

For example, for the product 78×78, you could write:

$78 \times 78 = (80 - 2)^2 = 80^2 - 2(80)(2) + 2^2 = 6400 - 320 + 4 = 6084$

Answers may vary. Samples:

1. 83×83
$(80 + 3)^2 = 80^2 + 2(80)(3) + 3^2 = 6889$

2. 63×57
$(60 + 3)(60 - 3) = 60^2 - 3^2 = 3591$

3. 59×59
$(60 - 1)^2 = 60^2 - 2(60)(1) + 1^2 = 3481$

4. 75×65
$(70 + 5)(70 - 5) = 70^2 - 5^2 = 4875$

5. 92×88
$(90 + 2)(90 - 2) = 90^2 - 2^2 = 8096$

6. 85×85
$(80 + 5)^2 = 80^2 + 2(80)(5) + 5^2 = 7225$

7. 71×71
$(70 + 1)^2 = 70^2 + 2(70)(1) + 1^2 = 5041$

8. 101×99
$(100 + 1)(100 - 1) = 100^2 - 1^2 = 9999$

9. 62×58
$(60 + 2)(60 - 2) = 60^2 - 2^2 = 3596$

10. 206×194
$(200 + 6)(200 - 6) = 200^2 - 6^2$
$= 39{,}964$

8-4 Think About a Plan

Multiplying Special Cases

Construction A square deck has a side length of $x + 5$. You are expanding the deck so that each side is four times as long as the side length of the original deck. What is the area of the new deck? Write your answer in standard form.

Understanding the Problem

1. What is the shape of the deck? square
2. How long is each side of the deck? $x + 5$
3. The new deck has sides that are 4 times longer than the original sides.
4. What is the problem asking you to find? area of new deck

Planning the Solution

5. Write an expression for the new side length of the deck.
   $4(x + 5)$, or $4x + 20$
6. Write an expression for the area of the new deck.
   $(4x + 20)^2$

Getting an Answer

7. What is the standard form of the expression for the area of the new deck?
   $16x^2 + 160x + 400$

8-4 Practice

Form G

Multiplying Special Cases

Simplify each expression.

1. $(x + 7)^2$

$x^2 + 14x + 49$

2. $(w + 9)^2$

$w^2 + 18w + 81$

3. $(h + 3)^2$

$h^2 + 6h + 9$

4. $(2s + 4)^2$

$4s^2 + 16s + 16$

5. $(3s + 1)^2$

$9s^2 + 6s + 1$

6. $(5s + 2)^2$

$25s^2 + 20s + 4$

7. $(a - 5)^2$

$a^2 - 10a + 25$

8. $(k - 10)^2$

$k^2 - 20k + 100$

9. $(n - 4)^2$

$n^2 - 8n + 16$

10. $(3m - 4)^2$

$9m^2 - 24m + 16$

11. $(6m - 2)^2$

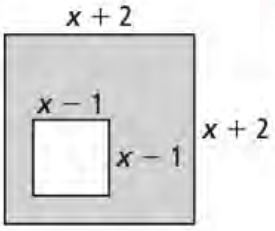
$36m^2 - 24m + 4$

12. $(4m - 2)^2$

$16m^2 - 16m + 4$

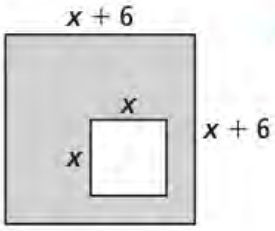
The figures below are squares. Find an expression for the area of each shaded region. Write your answers in standard form.

13.



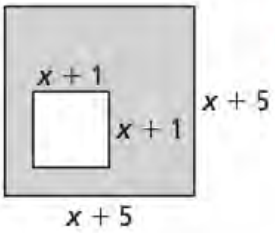
$6x + 3$

14.



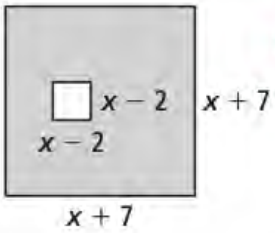
$12x + 36$

15.



$8x + 24$

16.



$18x + 45$

17. A square brown tarp has a square green patch green in the corner. The side length of the tarp is $(x + 8)$ and the side length of the patch is x. What is the area of the brown part of the tarp? $16x + 64$

18. A square red placemat has a gold square in the center. The side length of the gold square is $(x - 2)$ inches and the width of the red region is 4 inches. What is the area of the red part of the placemat? $-x^2 + 4x + 12$ square inches

8-4 Practice (continued)

Form G

Multiplying Special Cases

Mental Math **Simplify each product.**

19. 48^2 **2304**

20. 31^2 **961**

21. 29^2 **841**

22. 52^2 **2704**

23. 63^2 **3969**

24. 41^2 **1681**

25. 89^2 **7921**

26. 199^2 **39,601**

27. 302^2 **91,204**

Simplify each product.

28. $(v + 7)(v - 7)$
$v^2 - 49$

29. $(b + 2)(b - 2)$
$b^2 - 4$

30. $(z - 9)(z + 9)$
$z^2 - 81$

31. $(x + 12)(x - 12)$
$x^2 - 144$

32. $(8 + y)(8 - y)$
$64 - y^2$

33. $(t - 15)(t + 15)$
$t^2 - 225$

34. $(m + 1)(m - 1)$
$m^2 - 1$

35. $(a + 4)(a - 4)$
$a^2 - 16$

36. $(5 + g)(5 - g)$
$25 - g^2$

37. $(p + 20)(p - 20)$
$p^2 - 400$

38. $(f - 18)(f + 18)$
$f^2 - 324$

39. $(2c + 3)(2c - 3)$
$4c^2 - 9$

Mental Math **Simplify each product.**

40. $61 \cdot 59$
3599

41. $27 \cdot 33$
891

42. $202 \cdot 198$
39,996

43. $74 \cdot 66$
4884

44. $597 \cdot 603$
359,991

45. $85 \cdot 75$
6375

Simplify each product.

46. $(m + 4n)^2$
$m^2 + 8mn + 16n^2$

47. $(3a + b)^2$
$9a^2 + 6ab + b^2$

48. $(6s - t)^2$
$36s^2 - 12st + t^2$

49. $(s + 7t^2)^2$
$s^2 + 14st^2 + 49t^4$

50. $(p^5 - 8q^3)^2$
$p^{10} - 16p^5q^3 + 64q^6$

51. $(e^4 + f^2)^2$
$e^8 + 2e^4f^2 + f^4$

52. $(r^2 + 5s)(r^2 - 5s)$
$r^4 - 25s^2$

53. $(6p^2 + 2q)(6p^2 - 2q)$
$36p^4 - 4q^2$

54. $(3w^4 - z^3)(3w^4 + z^3)$
$9w^8 - z^6$

55. **Error Analysis** Describe and correct the error made in simplifying the product.
The x terms should have a sum of zero; $4x^2 - 49$

$(2x + 7)(2x - 7)$
$= 4x^2 - 28x - 49$

56. The formula $V = \frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r. Find the volume of a sphere with radius $x + 9$. Write your answer in standard form.
$V = \frac{4}{3}\pi x^3 + 36\pi x^2 + 324\pi x + 972\pi$

8-4 Practice

Form K

Multiplying Special Cases

Simplify each expression.

1. $(y + 1)^2$
$y^2 + 2y + 1$

2. $(n + 11)^2$
$n^2 + 22n + 121$

3. $(t + 7)^2$
$t^2 + 14t + 49$

4. $(3m + 6)^2$
$9m^2 + 36m + 36$

5. $(4x + 1)^2$
$16x^2 + 8x + 1$

6. $(3n + 2)^2$
$9n^2 + 12n + 4$

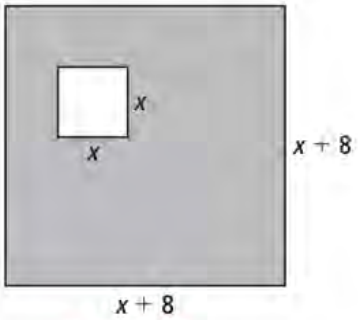
7. $(t - 3)^2$
$t^2 - 6t + 9$

8. $(7v - 3)^2$
$49v^2 - 42v + 9$

9. $(6p - 5)^2$
$36p^2 - 60p + 25$

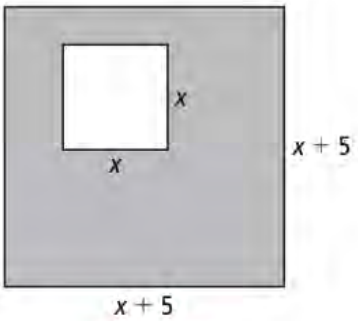
The figures below are squares. Find an expression for the area of each shaded region. Write your answers in standard form.

10.



$16x + 64$

11.



$10x + 25$

12. A flat, square roof needs a square patch in the corner to seal a leak. The side length of the roof is $(x + 12)$ ft and the side length of the patch is x ft. What is the area of the good part of the roof? $(24x + 144)$ ft

13. A white, square quilt has a purple square in the center. The side length of the purple square is $(x - 5)$ inches and the width of the quilt is 60 inches. What is the area of the white part of the quilt? $(-x^2 + 10x + 3575)$ in.^{2}

8-4 Practice (continued)

Form K

Multiplying Special Cases

Mental Math **Simplify each product.**

14. 52^2 **2704**

15. 18^2 **324**

16. 119^2 **14,161**

17. 495^2 **245,025**

18. 72^2 **5184**

19. 151^2 **22,801**

Simplify each product.

20. $(x + 1)(x - 1)$
$x^2 - 1$

21. $(m + 5)(m - 5)$
$m^2 - 25$

22. $(a - 4)(a + 4)$
$a^2 - 16$

23. $(s - 13)(s + 13)$
$s^2 - 169$

24. $(2z - 3)(2z + 3)$
$4z^2 - 9$

25. $(4d + 6)(4d - 6)$
$16d^2 - 36$

Mental Math **Simplify each product.**

26. $99 \cdot 101$ **9999**

27. $48 \cdot 52$ **2496**

28. $178 \cdot 182$ **32,396**

Simplify each product.

29. $(s + 3t)^2$
$s^2 + 6st + 9t^2$

30. $(2x + y)^2$
$4x^2 + 4xy + y^2$

31. $(4a - b)^2$
$16a^2 - 8ab + b^2$

32. $(m^2 + 3n)(m^2 - 3n)$
$m^4 - 9n^2$

33. $(9f^2 + 4g)(9f^2 - 4g)$
$81f^4 - 16g^2$

34. $(6m^4 - n^3)(6m^4 + n^3)$
$36m^8 - n^6$

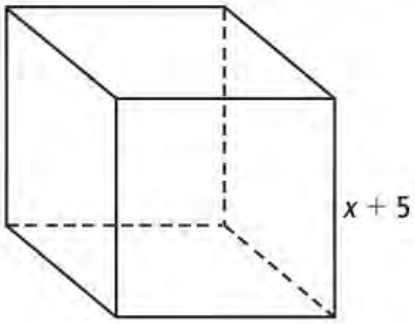
35. The formula $V = \pi r^2 h$ gives the volume of a cylinder with radius r and height h. Find the volume of a cylinder with radius $(x + 4)$ cm and height 5 cm. Write your answer in standard form. $(5\pi x^2 + 40\pi x + 80\pi)$ cm^3

8-4 Enrichment

Multiplying Special Cases

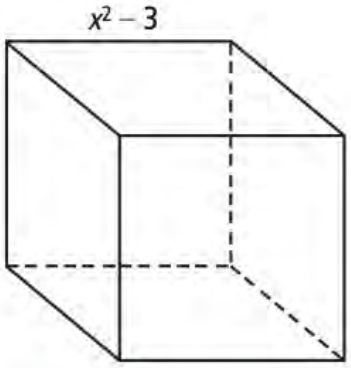
Find the volume of each cube.

1.



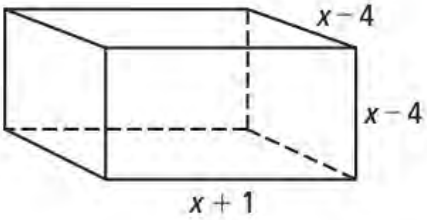
$x^3 + 15x^2 + 75x + 125$

2.



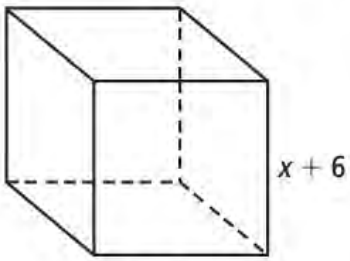
$x^6 - 9x^4 + 27x^2 - 27$

3. Find the volume of the rectangular prism.

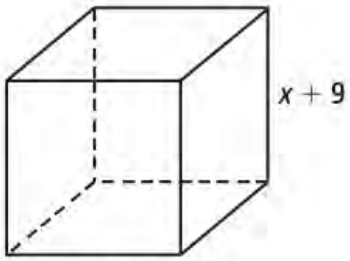


$x^3 - 7x^2 + 8x + 16$

4. How much greater is the volume of Cube B than the volume of Cube A?



Cube A



Cube B

$9x^2 + 135x + 513$

8-5 Additional Vocabulary Support

Factoring $x^2 + bx + c$

For Exercises 1–5, draw a line from each term in Column A to its definition in Column B. The first one is done for you.

$$(x + 4)(x + 8) = x^2 + (8 + 4)x + 4 \cdot 8 = x^2 + 12x + 32$$

Column A	Column B
1. 1	coefficient of trinomial's x^2 term
2. 12	binomial
3. 32	coefficient of trinomial's x term
4. $x + 4$	product of $(x + 4)$ and $(x + 8)$
5. $x^2 + 12x + 32$	trinomial's constant term

For Exercises 6–9, match the expression in Column A with its definition in Column B.

$$n^2 - 9n - 36 = (n - 12)(n + 3)$$

Column A	Column B
6. $(n - 12)(n + 3)$	factors of -36
7. $n^2 - 9n - 36$	sum of -12 and 3
8. -12 and 3	trinomial
9. -9	factored form of $n^2 - 9n - 36$

8-5 Reteaching

Factoring $x^2 + bx + c$

If a trinomial of the form $x^2 + bx + c$ can be written as the product of two binomials, then:

- The coefficient of the x-term in the trinomial is the sum of the constants in the binomials.
- The trinomial's constant term is the product of the constants in the binomials.

Problem

What is the factored form of $x^2 + 12x + 32$?

To write the factored form, you are looking for two factors of 32 that have a sum of 12.

Solve Make a table showing the factors of 32.

Factors of 32	Sum of Factors
1 and 32	33
2 and 16	18
4 and 8	12

$x^2 + 12x + 32 = (x + 4)(x + 8)$

Check $(x + 4)(x + 8)$

$x^2 + 8x + 4x + 32$ Use FOIL Method.

$x^2 + 12x + 32$ Combine the like terms.

Solution: The factored form of $x^2 + 12x + 32$ is $(x + 4)(x + 8)$.

Exercises

Factor each expression.

1. $x^2 + 9x + 20$
   $(x + 5)(x + 4)$
2. $y^2 + 12y + 35$
   $(y + 7)(y + 5)$
3. $z^2 + 8z + 15$
   $(z + 5)(z + 3)$
4. $a^2 + 11a + 28$
   $(a + 4)(a + 7)$
5. $b^2 + 10b + 16$
   $(b + 8)(b + 2)$
6. $c^2 + 12c + 27$
   $(c + 9)(c + 3)$
7. $d^2 + 6d + 5$
   $(d + 5)(d + 1)$
8. $e^2 + 15e + 54$
   $(e + 9)(e + 6)$
9. $f^2 + 11f + 24$
   $(f + 8)(f + 3)$

8-5 Reteaching (continued)

Factoring $x^2 + bx + c$

Some factorable trinomials in the form of $x^2 + bx + c$ will have negative coefficients. The rules for factoring are the same as when the x-term and the constant are positive.

- The coefficient of the x-term of the trinomial is the sum of the constants in the binomials.
- The trinomial's constant term is the product of the constants in the binomials.

However, one or both constants in the binomial factors will be negative.

Problem

What is the factored form of $x^2 - 3x - 40$?

To write the factored form, you are looking for two factors of -40 that have a sum of -3. The negative constant will have a greater absolute value than the positive constant.

Solve Make a table showing the factors of -40.

Factors of –40	Sum of Factors
1 and –40	–39
2 and –20	–18
4 and –10	–6
5 and –8	–3

$x^2 - 3x - 40 = (x - 8)(x + 5)$

Check $(x - 8)(x + 5)$

$x^2 + 5x - 8x - 40$ Use FOIL Method.

$x^2 + (-3x) - 40$ Combine the like terms.

Solution: The factored form of $x^2 - 3x - 40$ is $(x - 8)(x + 5)$.

Exercises

Factor each expression.

10. $s^2 + 2s - 35$
$(s + 7)(s - 5)$

11. $t^2 - 4t - 32$
$(t - 8)(t + 4)$

12. $u^2 + 6u - 27$
$(u + 9)(u - 3)$

13. $v^2 - 2v + 48$
$(v - 8)(v + 6)$

14. $w^2 - 8w - 9$
$(w - 9)(w + 1)$

15. $x^2 + 3x - 18$
$(x + 6)(x - 3)$

8-5 Think About a Plan

Factoring $x^2 + bx + c$

Recreation A rectangular skateboard park has an area of $x^2 + 15x + 54$. What are possible dimensions of the park? Use factoring.

Know

1. The area of the skateboard park is $x^2 + 15x + 54$.
2. The dimensions of a rectangle are its length and width.
3. The factors of the area polynomial are possible dimensions of the skateboard park.

Need

4 To solve the problem I need to find the factors of $x^2 + 15x + 54$

Plan

5. Complete the table. List the pairs of factors of $\boxed{54}$.

   Identify the pair that has a sum of $\boxed{15}$.

Factors of 54	Sum of Factors

Factors	Sum
1 and 54	55
2 and 27	29
3 and 18	21
6 and 9	15

6. Write the factored polynomial.

   $(x + 6)(x + 9)$

7. What are possible dimensions of the skateboard park?

   length: $x + 9$; width $x + 6$

8. Justify your answer.

   If the length is $x + 9$ and the width is $x + 6$, then the area is $(x + 9)(x + 6) = x^2 + 15x + 54$.

8-5 Practice

Form G

Factoring $x^2 + bx + c$

Complete.

1. $k^2 + 11k + 30 = (k + 5)(k + \boxed{6})$
2. $x^2 + 6x + 9 = (x + 3)(x + \boxed{3})$
3. $t^2 + 7t + 10 = (t + 2)(t + \boxed{5})$
4. $n^2 + 9n + 14 = (n + 7)(n + \boxed{2})$
5. $w^2 + 13w + 36 = (w + 4)(w + \boxed{9})$
6. $y^2 + 18y + 65 = (y + 13)(y + \boxed{5})$
7. $s^2 - 12s + 32 = (s - 8)(s - \boxed{4})$
8. $g^2 - 14g + 45 = (g - 9)(g - \boxed{5})$
9. $v^2 - 17v + 60 = (v - 12)(v - \boxed{5})$
10. $q^2 - 13q + 42 = (q - 6)(q - \boxed{7})$
11. $d^2 - 9d + 8 = (d - 8)(d - \boxed{1})$
12. $r^2 - 9r + 20 = (r - 5)(r - \boxed{4})$

Factor each expression. Check your answer.

13. $y^2 + 5y + 6$
    $(y + 3)(y + 2)$
14. $t^2 + 9t + 18$
    $(t + 6)(t + 3)$
15. $x^2 + 16x + 63$
    $(x + 9)(x + 7)$
16. $n^2 - 12n + 35$
    $(n - 7)(n - 5)$
17. $r^2 - 12r + 27$
    $(r - 9)(r - 3)$
18. $q^2 - 12q + 20$
    $(q - 10)(q - 2)$
19. $w^2 + 19w + 60$
    $(w + 15)(w + 4)$
20. $b^2 - 11b + 24$
    $(b - 8)(b - 3)$
21. $z^2 - 13z + 12$
    $(z - 12)(z - 1)$

Complete.

22. $q^2 + q - 56 = (q - 7)(q + \boxed{8})$
23. $z^2 - 3z - 18 = (z - 6)(z + \boxed{3})$
24. $n^2 - 6n - 40 = (n + 4)(n - \boxed{10})$
25. $y^2 + 3y - 4 = (y + 4)(y - \boxed{1})$
26. $v^2 - 5v - 36 = (v - 9)(v + \boxed{4})$
27. $d^2 + 2d - 15 = (d - 3)(d + \boxed{5})$
28. $m^2 - 5m - 14 = (m + 2)(m - \boxed{7})$
29. $p^2 - 6p - 16 = (p - 8)(p + \boxed{2})$

8-5 Practice (continued)

Form G

Factoring $x^2 + bx + c$

Factor each expression. Check your answer.

30. $r^2 + 3r - 10$
$(r + 5)(r - 2)$

31. $w^2 + 2w - 8$
$(w + 4)(w - 2)$

32. $z^2 + 3z - 40$
$(z + 8)(z - 5)$

33. $d^2 - 4d - 12$
$(d - 6)(d + 2)$

34. $p^2 - 7p - 8$
$(p - 8)(p + 1)$

35. $s^2 - 5s - 24$
$(s - 8)(s + 3)$

36. $x^2 + 5x - 6$
$(x + 6)(x - 1)$

37. $v^2 + 3v - 28$
$(v + 7)(v - 4)$

38. $n^2 + 2n - 63$
$(n + 9)(n - 7)$

39. $t^2 - 2t - 24$
$(t - 6)(t + 4)$

40 $a^2 - 7a - 18$
$(a - 9)(a + 2)$

41. $c^2 - c - 30$
$(c - 6)(c + 5)$

42. The area of a rectangular door is given by the trinomial $x^2 - 14x + 45$. The door's width is $(x - 9)$. What is the door's length? $x - 5$

43. The area of a rectangular painting is given by the trinomial $a^2 - 6a - 16$. The painting's length is $(a + 2)$. What is the painting's width? $a - 8$

Write the correct factored form for each expression.

44. $k^2 + 4kn - 96n^2$
$(k + 12n)(k - 8n)$

45. $g^2 - 13gh + 42h^2$
$(g - 6h)(g - 7h)$

46. $m^2 - 4mn - 32n^2$
$(m - 8n)(m + 4n)$

47. $x^2 + 5xy - 14y^2$
$(x + 7y)(x - 2y)$

48. $s^2 + 17st + 72t^2$
$(s + 8t)(s + 9t)$

49. $h^2 + 3hj - 88j^2$
$(h + 11j)(h - 8j)$

50. Error Analysis Describe and correct the error made in factoring the trinomial.

$\cancel{x^2 + 2x - 80 = (x + 8)(x - 10)}$

The operation signs are wrong. The answer should be $(x - 8)(x + 10)$.

51. A rectangular pool cover has an area of $p^2 + 9p - 36$. What are possible dimensions of the pool cover? Use factoring.

$(p + 12)$ and $(p - 3)$

8-5 Practice

Form K

Factoring $x^2 + bx + c$

Complete.

1. $n^2 + 9n + 18 = (n + 3)(n + \boxed{6})$
2. $t^2 + 9t + 14 = (t + 2)(t + \boxed{7})$
3. $d^2 + 11d + 30 = (d + 5)(d + \boxed{6})$
4. $v^2 + 2v + 1 = (v + 1)(v + \boxed{1})$
5. $m^2 - 8m + 15 = (m - 5)(m - \boxed{3})$
6. $a^2 - 13a + 22 = (a - 2)(a - \boxed{11})$
7. $z^2 - 17z + 72 = (z - 8)(z - \boxed{9})$
8. $w^2 - 7w + 12 = (w - 3)(w - \boxed{4})$

Factor each expression. Check your answer.

9. $g^2 + 6g + 8$
   $(g + 2)(g + 4)$
10. $y^2 + 10y + 24$
    $(y + 6)(y + 4)$
11. $r^2 + 12r + 35$
    $(r + 5)(r + 7)$
12. $k^2 + 9k + 8$
    $(k + 1)(k + 8)$
13. $x^2 - 16x + 60$
    $(x - 10)(x - 6)$
14. $h^2 - 19h + 78$
    $(h - 13)(h - 6)$

Complete.

15. $g^2 + 5g - 24 = (g - 3)(g + \boxed{8})$
16. $b^2 - 6b - 7 = (b - 7)(b + \boxed{1})$
17. $y^2 + 4y - 45 = (y + 9)(y - \boxed{5})$
18. $k^2 + 4k - 12 = (k + 6)(k - \boxed{2})$
19. $p^2 - 7p - 60 = (p + 5)(p - \boxed{12})$
20. $n^2 - 6n - 40 = (n - 10)(n + \boxed{4})$

8-5 Practice (continued)

Form K

Factoring $x^2 + bx + c$

Factor each expression. Check your answer.

21. $x^2 - 4x - 5$
$(x + 1)(x - 5)$

22. $t^2 + t - 20$
$(t + 5)(t - 4)$

23. $z^2 - z - 72$
$(z + 8)(z - 9)$

24. $m^2 - 6m - 27$
$(m + 3)(m - 9)$

25. $a^2 + 4a - 21$
$(a + 7)(a - 3)$

26. $v^2 - 4v - 12$
$(v + 2)(v - 6)$

27. $c^2 - 7c - 44$
$(c + 4)(c - 11)$

28. $r^2 + 6r - 16$
$(r + 8)(r - 2)$

29. $f^2 + f - 6$
$(f + 3)(f - 2)$

30. $j^2 - 6j - 55$
$(j + 5)(j - 11)$

31. $y^2 + 3y - 54$
$(y + 9)(y - 6)$

32. $n^2 - 10n - 11$
$(n + 1)(n - 11)$

33. The area of a rectangular window is given by the trinomial $x^2 - 14x + 48$. The window's length is $(x - 8)$. What is the window's width? $(x - 6)$

34. The area of a rectangular area rug is given by the trinomial $f^2 - 4f - 77$. The length of the rug is $(f + 7)$. What is the width of the rug? $(f - 11)$

35. Reasoning Write possible expressions for the length and the width of a rectangle with area $x^2 + 13x + 42$. $(x + 6)$; $(x + 7)$

36. A rectangular tabletop has an area of $t^2 + 2t - 99$. What are possible dimensions of the tabletop? Use factoring. $t + 11$ and $t - 9$

8-5 Enrichment

Factoring $x^2 + bx + c$

To factor a trinomial of the form $x^2 + bx + c$ as the product of binomials, you must find factor pairs that have a sum of b and a product of c. Examine what happens to c as you increase b when c is greater than zero.

If $b = 2$, the factor pair is 1, 1 and the product is 1. $(x + 1)(x + 1) = x^2 + 2x + 1$

If $b = 3$, the factor pair is 1, 2 and the product is 2. $(x + 1)(x + 2) = x^2 + 3x + 2$

If $b = 4$, the factor pairs are 1, 3 and 2, 2. The products are 3 and 4.

$(x + 1)(x + 3) = x^2 + 4x + 3$ $\quad$ $(x + 2)(x + 2) = x^2 + 4x + 4$

If $b = 5$, the factor pairs are 1, 4 and 2, 3. The products are 4 and 6.

$(x + 1)(x + 4) = x^2 + 5x + 4$ $\quad$ $(x + 2)(x + 3) = x^2 + 5x + 6$

1. What are the factor pairs and products (values of c) for the following values of b, for $x^2 + bx + c$ if $c > 0$?

   $b = 6$ $\quad$ $b = 7$ $\quad$ $b = 8$ $\quad$ $b = 9$ $\quad$ $b = 10$

   $b = 6$; pairs: 1,5; 2, 4; 3, 3; products: 5, 8, 9 $b = 7$; pairs: 1, 6; 2, 5; 3, 4; products: 6, 10, 12 $b = 8$; pairs: 1,7; 2, 6; 3, 5; 4, 4; products: 7, 12, 15, 16 $b = 9$; pairs: 1, 8; 2, 7; 3, 6; 4, 5; products: 8, 14, 18, 20 $b = 10$; pairs: 1, 9; 2, 8; 3, 7; 4, 6; 5, 5; products: 9, 16, 21, 24, 25

2. What pattern do you see in the number of factor pairs (and thus values for c) as you increase the value of b?

   The number of factor pairs increases as b increases

3. Describe at least one pattern you see in the value of c in terms of b.

   Answers may vary. Sample: If b is even then $\frac{b}{2}$ is the number of c values. If b is odd, then $\frac{(b-1)}{2}$ is the number of c values.

Now examine what happens to the value of b when the value of c changes, when $c > 0$.

If $c = 1$, the factor pair is 1, 1, and the sum is 2. $(x + 1)(x + 1) = x^2 + 2x + 1$

If $c = 2$, the factor pair is 1, 2 and the sum is 3. $(x + 1)(x + 2) = x^2 + 3x + 2$

If $c = 3$, the factor pair is 1, 3 and the sum is 4. $(x + 1)(x + 3) = x^2 + 4x + 3$

If $c = 4$, the factors pairs are 1, 4 and 2, 2. The sums are 5 and 4.

$(x + 1)(x + 4) = x^2 + 5x + 4$ $\quad$ $(x + 2)(x + 2) = x^2 + 4x + 4$

4. What are the factor pairs and sums (values of b) for the following values of c, for $x^2 + bx + c$ if $c > 0$?

   $c = 5$ $\quad$ $c = 6$ $\quad$ $c = 7$ $\quad$ $c = 8$

   $c = 5$; pairs: 1, 5; sums: 6 $\quad$ $c = 6$; pairs: 1, 6; 2, 3; sums: 7, 5
   $c = 7$; pairs: 1, 7; sums: 8 $\quad$ $c = 8$; pairs: 1, 8; 2, 4; sums: 9, 6

5. Describe at least one pattern you see in the value of b in terms of c. Explain why this might be.

   Answers may vary. Sample: Prime numbers have only one pair of factors because the factors of a prime number are the number and 1.

8-6 Additional Vocabulary Support

Factoring $ax^2 + bx + c$

A student is trying to factor $3x^2 + 13x + 4$. She wrote these steps to solve the problem on note cards, but they got mixed up.

Find factors of *ac* that have sum *b*.

$(3x + 1)(x + 4)$

Since ac = 12 and b = 13, find positive factors of 12 that have sum 13.

To factor the trinomial, use the factors you found to rewrite *bx* as 1x + 12x.

Make a table.

Factors of 12	2, 6	3, 4	1, 12
Sum of factors	8	7	13 ✓

Use the note cards to complete the steps below.

1. First, find factors of *ac* that have sum *b*.

2. Second, since *ac* = 12 and *b* = 13, find positive factors of 12 that have sum 13.

3. Third,

Factors of 12	2, 6	3, 4	1, 12
Sum of factors	8	7	13 ✓

4. Then, to factor the trinomial, use the factors you found to rewrite *bx*.

5. Finally, $(3x + 1)(x + 4)$

8-6 Reteaching

Factoring $ax^2 + bx + c$

You can use your knowledge of prime numbers to help you factor some trinomials as two binomials. A prime number has only 1 and itself as factors. For trinomials of the form $ax^2 + bx + c$, if a is a prime number then you already know the first term of each binomial: ax and $1x$. Then list the factors that will multiply to produce c. Use guess and check to find the factor pair that will add to b.

Problem

What is the factored form of $7x^2 + 31x + 12$?

$7x^2 + 31x + 12 = (7x \quad)(1x \quad)$ — a is 7, which is prime, so the factors are 7 and 1.

$= (7x \quad)(x \quad)$ — You don't need the 1 in front of the variable, so drop it.

$7x^2 + 31x + 12 = (7x + \quad)(x + \quad)$ — The trinomial has two plus signs, so the binomials also have plus signs.

Because c is 12, find factor pairs that multiply to 12: (1 and 12), (2 and 6), (3 and 4).

Try each pair in the expression to see if the INNER and OUTER products add to b, or 31.

$(7x + 1)(x + 12) = 7x^2 + x + 84x = 7x^2 + \mathbf{85}x + 12$ (NO)

$(7x + 2)(x + 6) = 7x^2 + 2x + 42x = 7x^2 + \mathbf{44}x + 12$ (NO)

$(7x + 3)(x + 4) = 7x^2 + 3x + 28x = 7x^2 + \mathbf{31}x + 12$ (YES)

The factored form of $7x^2 + 31x + 12$ is $(7x + 3)(x + 4)$.

Exercises

Factor each expression.

1. $3x^2 + 14x + 8$
   $(3x + 2)(x + 4)$
2. $5y^2 + 43y + 24$
   $(5y + 3)(y + 8)$
3. $2z^2 + 19z + 42$
   $(2z + 7)(z + 6)$
4. $11a^2 + 39a + 18$
   $(11a + 6)(a + 3)$
5. $13b^2 + 58b + 24$
   $(13b + 6)(b + 4)$
6. $23c^2 + 56c + 20$
   $(23c + 10)(c + 2)$
7. $7d^2 + d - 8$
   $(7d + 8)(d - 1)$
8. $3e^2 + 20e - 32$
   $(3e - 4)(e + 8)$
9. $19f^2 + 10f - 9$
   $(19f - 9)(f + 1)$
10. $5s^2 - 18s + 16$
    $(5s - 8)(s - 2)$
11. $17t^2 - 12t - 5$
    $(17t + 5)(t - 1)$
12. $29u^2 + 48u - 20$
    $(29u - 10)(u + 2)$

8-6 Reteaching (continued)

Factoring $ax^2 + bx + c$

If you are given the area and one side of a rectangle, you can find the second side by factoring the trinomial. One binomial is the width and the other binomial is the length.

Problem

The area of a rectangular swimming pool is $6x^2 + 11x + 3$. The width of the pool is $2x + 3$. What is the length of the pool?

You are given the area and length of the pool. Set up an equation with what you are given and solve or length.

$6x^2 + 11x + 3 = (2x + 3)(\square\square\square)$	Area = length × width.
$6x^2 + 11x + 3 = (2x + 3)(3x\,\square\square)$	$6x^2 = (2x)(3x)$, so the first term of the second binomial is $3x$.
$6x^2 + 11x + 3 = (2x + 3)(3x + \square)$	The trinomial has two plus signs, so the sign for the second binomial must also be plus.
$6x^2 + 11x + 3 = (2x + 3)(3x + 1)$	The value of c is 3. Since $3 = 3 \times 1$, the second term must be 1.

Multiply to check your answer. Use FOIL.

$(2x + 3)(3x + 1) = 6x^2 + 2x + 9x + 3 = 6x^2 + 11x + 3$ ✓

The length of the swimming pool is $3x + 1$.

Exercises

13. The area of a rectangular cookie sheet is $8x^2 + 26x + 15$. The width of the cookie sheet is $2x + 5$. What is the length of the cookie sheet?

$4x + 3$

14. The area of a rectangular lobby floor in the new office building is $15x^2 + 47x + 28$. The length of one side of the lobby is $5x + 4$. What is the width?

$3x + 7$

15. The area of a rectangular school banner is $12x^2 + 13x - 90$. The width of the banner is $3x + 10$. What is the length of the banner?

$4x - 9$

16. The distance a train has traveled is $6x^2 - 23x + 20$. The train's average speed is $3x - 4$. How long has the train been traveling?

$2x - 5$

8-6 Think About a Plan

Factoring $ax^2 + bx + c$

Carpentry The top of a rectangular table has an area of $18x^2 + 69x + 60$. The width of the table is $3x + 4$. What is the length of the table?

Know

1. The area of the table top is $18x^2 + 69x + 60$.
2. The width of the table top is $3x + 4$.
3. Some quadratic trinomials can be written as the product of two binomials.
4. One of the factors of the polynomial $18x^2 + 69x + 60$ is $3x + 4$.

Need

5. To solve the problem I need to find the other factor.

Plan

6. Find the missing factor.

   What can you multiply by $3x$ to get $18x^2$? $3x \cdot \boxed{6x} = 18x^2$

   What can you multiply by 4 to get 60? $4 \cdot \boxed{15} = 60$

7. What is the factored form of $18x^2 + 69x + 60$? $(3x + 4)(6x + 15)$

8. What is the length of the table? Check your answer.

   length: $(6x + 15)$

   Check: $(3x + 4)(6x + 15) = 18x^2 + 69x + 60$

8-6 Practice

Form G

Factoring $ax^2 + bx + c$

Factor each expression.

1. $2w^2 + 13w + 15$
   $(2w + 3)(w + 5)$
2. $3d^2 + 20d + 12$
   $(3d + 2)(d + 6)$
3. $4n^2 + 62n - 32$
   $2(2n - 1)(n + 16)$
4. $3p^2 - 7p - 40$
   $(3p + 8)(p - 5)$
5. $6r^2 - 10r - 24$
   $2(3r + 4)(r - 3)$
6. $5z^2 - 17z + 14$
   $(5z - 7)(z - 2)$
7. $14k^2 - 67k + 63$
   $(2k - 7)(7k - 9)$
8. $2m^2 - m - 15$
   $(2m + 5)(m - 3)$
9. $3x^2 + 9x - 84$
   $3(x + 7)(x - 4)$
10. $4y^2 + 26y + 30$
    $2(2y + 3)(y + 5)$
11. $5t^2 - 24t - 5$
    $(5t + 1)(t - 5)$
12. $7c^2 - 2c - 9$
    $(7c - 9)(c + 1)$
13. $8k^2 - 42k + 27$
    $(4k - 3)(2k - 9)$
14. $6g^2 - 2g - 20$
    $2(3g + 5)(g - 2)$
15. $2c^2 - 23c + 11$
    $(2c - 1)(c - 11)$
16. The area of a rectangular computer screen is $4x^2 + 20x + 16$. The width of the screen is $2x + 8$. What is the length of the screen?
    $2x + 2$
17. The area of a rectangular granite countertop is $12x^2 + 10x - 12$. The width of the countertop is $2x + 3$. What is the length of the countertop?
    $6x - 4$
18. The area of a rectangular book cover is $4x^2 - 6x - 40$. The width of the book cover is $2x - 8$. What is the length of the book cover?
    $2x + 5$
19. The area of a rectangular parking lot is $21x^2 - 44x + 15$. The width of the parking lot is $3x - 5$. What is the length of the parking lot?
    $7x - 3$

Factor each expression completely.

20. $6x^2 - 10x - 4$
    $2(3x + 1)(x - 2)$
21. $6d^2 + 21d + 15$
    $3(2d + 5)(d + 1)$
22. $8n^2 + 68n + 84$
    $4(2n + 3)(n + 7)$
23. $20p^2 - 115p - 30$
    $5(4p + 1)(p - 6)$
24. $15r^2 + 141r - 90$
    $3(5r - 3)(r + 10)$
25. $12z^2 - 14z + 4$
    $2(2z - 1)(3z - 2)$
26. $20k^2 + 110k + 120$
    $10(2k + 3)(k + 4)$
27. $9m^2 - 66m + 21$
    $3(3m - 1)(m - 7)$
28. $40x^2 - 136x - 96$
    $8(5x + 3)(x - 4)$
29. $42y^2 + 28y - 14$
    $14(3y - 1)(y + 1)$
30. $8t^2 - 16t - 90$
    $2(2t + 5)(2t - 9)$
31. $24c^2 + 96c + 90$
    $6(2c + 5)(2c + 3)$

8-6 Practice (continued)

Form G

Factoring $ax^2 + bx + c$

Open-Ended **Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.**

32. $4x^2 + \square x + 12$

Answers may vary. Sample: 19, 16: $(4x + 3)(x + 4)$; $(4x + 4)(x + 3)$

33. $6t^2 - \square t - 4$

Answers may vary. Sample: 23, −5; $(6t + 1)(t - 4)$; $(3t + 4)(2t - 1)$

34. $9m^2 - \square m + 8$

Answers may vary. Sample: 73, 27; $(9m - 1)(m - 8)$; $(3m - 8)(3m - 1)$

35. $8n^2 + \square n - 10$

Answers may vary. Sample: 11, −11; $(8n - 5)(n + 2)$; $(n - 2)(8n + 5)$

36. $12v^2 - \square v + 15$

Answers may vary. Sample: 29, 27; $(4v - 3)(3v - 5)$; $(4v - 5)(3v - 3)$

37. $5w^2 - \square w - 24$

Answers may vary. Sample. 26, 14; $(5w + 4)(w - 6)$; $(5w + 6)(w - 4)$

38. Error Analysis Describe and correct the error made in factoring the expression at the right.

$$\begin{aligned}(6x^2 + 3x - 9) &= 3(2x^2 + x - 3)\\ &= 3(2x^2 - 3x + 2x - 3)\\ &= 3(2x^2 - 3x + (2x - 3)\\ &= 3[\,x\,(2x - 3) + 1\,(2x - 3)\,]\\ &= 3(x + 1)\,(2x - 3)\end{aligned}$$

In the second step, the student wrote $-1x$ instead of $1x$. x should be written as $3x - 2x$. Answer: $3(2x + 3)(x - 1)$

39. A parallelogram has an area of $4x^2 + 7x - 15$. The base of the parallelogram is $x + 3$. What is the height of the parallelogram?

a. Write the formula for the area of a parallelogram. **$A = bh$**

b. Writing Explain how factoring the trinomial helps you solve the problem.

Factor to find h: $(x + 3)(4x - 5) = 4x^2 + 7x - 15$; $h = 4x - 5$

40. A rectangular window pane has an area of $15x^2 - 19x + 6$. The width of the window pane is $3x - 2$. What is the length of the window pane?

$5x - 3$

Factor each expression completely.

41. $28y^2 + 43y - 48$

$(4y - 3)(7y + 16)$

42. $16z^2 - 54z + 35$

$(8z - 7)(2z - 5)$

43. $27n^2 - 54n + 15$

$3(3n - 1)(3n - 5)$

44. $36p^2 + 63p + 20$

$(3p + 4)(12p + 5)$

45. $28r^2 - 20r - 33$

$(2r - 3)(14r + 11)$

46. $30z^2 - 53z + 12$

$(2z - 3)(15z - 4)$

47. $32x^3 + 28x^2 + 5x$

$x(4x + 1)(8x + 5)$

48. $25p^2 + 20pq - 12q^2$

$(5p - 2q)(5p + 6q)$

49. $72g^2h - 43gh + 6h$

$h(9g - 2)(8g - 3)$

8-6 Practice

Form K

Factoring $ax^2 + bx + c$

Factor each expression.

1. $3n^2 - 8n - 3$
$(3n + 1)(n - 3)$

2. $5a^2 - 22a + 8$
$(5a - 2)(a - 4)$

3. $2s^2 + 13s + 6$
$(2s + 1)(s + 6)$

4. $6t^2 + 21t - 12$
$3(2t - 1)(t + 4)$

5. $9b^2 - 65b + 14$
$(9b - 2)(b - 7)$

6. $5z^2 + 11z + 6$
$(5z + 6)(z + 1)$

7. $7r^2 - 9r - 10$
$(7r + 5)(r - 2)$

8. $2m^2 + m - 21$
$(2m + 7)(m - 3)$

9. $3g^2 + 20g + 32$
$(3g + 8)(g + 4)$

10. The area of a rectangular driveway is $2x^2 + 15x + 25$. The width of the driveway is $x + 5$. What is the length of the driveway? $2x + 5$

11. The area of a rectangular floor is $8x^2 + 6x - 20$. The width of the floor is $2x + 4$. What is the length of the floor? $4x - 5$

12. The area of a rectangular desktop is $6x^2 - 3x - 3$. The width of the desktop is $2x + 1$. What is the length of the desktop? $3x - 3$

Factor each expression completely.

13. $24n^2 + 2n - 12$
$2(4n + 3)(3n - 2)$

14. $72q^2 - 12q - 40$
$4(3q + 2)(6q - 5)$

15. $30j^2 - 27j - 21$
$3(2j + 1)(5j - 7)$

16. $60h^2 + 280h + 45$
$5(6h + 1)(2h + 9)$

17. $40a^2 + 126a + 44$
$2(4a + 11)(5a + 2)$

18. $45f^2 + 24f - 189$
$3(5f - 9)(3f + 7)$

8-6 Practice (continued)

Form K

Factoring $ax^2 + bx + c$

Open-Ended **Find two different values that complete each expression so that the trinomial can be factored into the product of two binomials. Factor your trinomials.**

19. $4n^2 + \square n - 3$

Answers may vary. Sample: $-4, 11$; $4n^2 - 4n - 3 = (2n - 3)(2n + 1)$; $4n^2 + 11n - 3 = (n + 3)(4n - 1)$

20. $12r^2 + \square + 6$

Answers may vary. Sample: $17, 38$; $12r^2 + 17r + 6 = (3r + 2)(4r + 3)$; $12r^2 + 38r + 6 = (2r + 6)(6r + 1)$

21. $24a^2 + \square a - 15$

Answers may vary. Sample: $-18, 37$; $24a^2 - 18a - 15 = (6a + 3)(4a - 5)$; $24a^2 + 37a - 15 = (8a + 15)(3a - 1)$

22. $18b^2 + \square b + 8$

Answers may vary. Sample: $24, 74$; $18b^2 + 24b + 8 = (3b + 2)(6b + 4)$; $18b^2 + 74b + 8 = (9b + 1)(2b + 8)$

23. A parallelogram has an area of $8x^2 - 2x - 45$. The height of the parallelogram is $4x + 9$.

a. Write the formula for the area of a parallelogram. **$A = bh$**

b. What is the length of the base of the parallelogram? **$2x - 5$**

c. **Writing** Explain how you solved the problem.

Sample: You know that the product of $4x + 9$ and another factor is $8x^2 - 2x - 45$. $4x$ times $2x$ is $8x^2$ and 9 times -5 is -45. So, $8x^2 - 2x - 45 = (4x + 9)(2x - 5)$. Then use FOIL to check.

24. A rectangular athletic field has an area of $40x^2 + 190x - 50$. The width of the athletic field is $8x - 2$. What is the length of the athletic field? **$5x + 25$**

Factor each expression.

25. $96d^2 - 76d - 77$

$(12d + 7)(8d - 11)$

26. $48h^2 - 86h + 35$

$(8h - 5)(6h - 7)$

27. $24m^2 + 18m - 15$

$3(2m - 1)(4m + 5)$

28. $36c^2 + 27c - 55$

$(3c + 5)(12c - 11)$

8-6 Enrichment

Factoring $ax^2 + bx + c$

You can use a function to estimate the volume of an adult body based on the length of one part, such as the length of an index finger, x. Start by using x to calculate the volume of an index finger. Assume the ratio of the length to height to width of an average index finger is 7 : 1 : 2. Therefore, the volume is $\frac{2}{49}x^3$.

You can then estimate that approximately 10 index fingers make up one hand. Multiply the volume of one index finger by 10 to find the volume in one hand: $\frac{20}{49}x^3$. Use this more convenient hand measure to figure out how many hands make up each large body area.

Hand = 1 hand	Arm ≈ 12 hands	Head ≈ 12 hands	Neck ≈ 8 hands
Torso ≈ 100 hands	Leg ≈ 45 hands	Foot ≈ 3 hands	

Add up all the parts, making sure to double the hands, arms, and legs:
$2 + 24 + 12 + 8 + 100 + 90 + 6 = 242$ hands
Now multiply the number of hands by the volume in one hand:
$242\left(\frac{20}{49}\right)x^3 = \frac{4840}{49}x^3 = 98\frac{38}{49}x^3$

Now that you have a function for the volume of a human body, you can use it to find expressions for other body parts without measuring.

1. Use the function $V = 98\frac{38}{49}x^3$, the volume of an adult body to write an expression for the length of the foot in an adult body. where the ratio of the length to height to width of the foot is 6 : 1 : 1.

   $\frac{60}{49}x$

2. Use the same function to write an expression for the length of an arm in an adult body where the ratio of the length to height to width of the arm is 10 : 1 : 1.

   $\frac{240}{49}x$

3. Measure the lengths of three people's index fingers, feet, and arms. How do the results compare to your estimates?

   Check students' work.

8-7 Additional Vocabulary Support

Factoring Special Cases

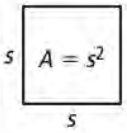
Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
difference of two squares	A binomial in which a perfect square monomial is subtracted from another perfect square monomial	$x^2 - 16$
factoring a difference of two squares	1. To factor the difference of two squares a^2 and b^2, multiply the sum of the two factors a and b by the difference of the two factors a and b. $a^2 - b^2 = (a + b)(a - b)$	$x^2 - 25 = (x + 5)(x - 5)$
factoring perfect-square trinomials	For every real number a and b: $a^2 + 2ab + b^2 =$ $(a + b)(a + b) = (a + b)^2$ or $a^2 - 2ab + b^2 =$ $(a - b)(a - b) = (a - b)^2$.	2. $4x^2 - 20x + 25 =$ $(2x - 5)(2x - 5)$
perfect-square trinomial	3. Any trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ is a *perfect-square trinomial* because it is the result of squaring a binomial.	$9x^2 + 24x + 16$

8-7 Reteaching

Factoring Special Cases

The area of a square is given by $A = s^2$, where s is a side length. When the side length is a binomial, the area can be written as a perfect-square trinomial. If you are given the area of such a square, you can use factoring to write an expression for a side length.



Problem

A mosaic is made of small square tiles called tesserae. Suppose the area of one tessera is $9x^2 + 12x + 4$. What is the length of one side of a tessera?

Because the tile is a square, you know the side lengths must be equal. Therefore, the binomial factors of the trinomial must be equal.

$9x^2 + 12x + 4 = (\square\square\square)^2$	This is a perfect square trinomial and can be factored as the square of a binomial.
$9x^2 = (3x)^2$	$9x^2$ and 4 are perfect squares. Write them as squares.
$4 = 2^2$	
$2(3x)(2) = 12x$	Check that $12x$ is twice the product of the first and last terms. It is, so you are sure that you have a perfect-square trinomial.
$9x^2 + 12x + 4 = (3x + 2)^2$	Rewrite the equation as the square of a binomial.

Multiply to check your answer.

$$(3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4 = 9x^2 + 12x + 4 ✓$$

The length of one side of the square is $3x + 2$.

Exercises

Factor each expression to find the side length.

1. The area of a square oil painting is $4x^2 + 28x + 49$. What is the length of one side of the painting?

   $2x + 7$

2. You are installing linoleum squares in your kitchen. The area of each linoleum square is $16x^2 - 24x + 9$. What is the length of one side of a linoleum square?

   $4x - 3$

3. You are building a table with a circular top. The area of the tabletop is $(25x^2 - 40x + 16)\pi$. What is the radius of the tabletop?

   $5x - 4$

4. A fabric designer is making a checked pattern. Each square in the pattern has an area of $x^2 - 16x + 64$. What is the length of one side of a check?

   $x - 8$

8-7 Reteaching (continued)

Factoring Special Cases

Some binomials are a difference of two squares. To factor these expressions, write the factors so the x-terms cancel and you are left with two perfect squares.

Problem

What is the factored form of $4x^2 - 9$?

$4x^2 - 9 = (\square + \square)(\square - \square)$	Both $4x^2$ and 9 are perfect squares. You know the signs of the factors will be opposite, so the x-terms will cancel out.
$\sqrt{4x^2} = 2x$	Find the square root of each term.
$\sqrt{9} = 3$	
$(2x + 3)(2x - 3)$	Write each term as a binomial with opposite signs, so the x-terms will cancel out.

Multiply to check your answer.

$$(2x + 3)(2x - 3) = 4x^2 + 6x - 6x - 9$$
$$= 4x^2 - 9 ✓$$

The factored form of $4x^2 - 9$ is $(2x + 3)(2x - 3)$.

Exercises

Factor each expression.

5. $9x^2 - 4$
$(3x + 2)(3x - 2)$

6. $25x^2 - 49$
$(5x + 7)(5x - 7)$

7. $144x^2 - 1$
$(12x + 1)(12x - 1)$

8. $64x^2 - 25$
$(8x + 5)(8x - 5)$

9. $49x^2 - 16$
$(7x + 4)(7x - 4)$

10. $36x^2 - 49$
$(6x + 7)(6x - 7)$

11. $81x^2 - 16$
$(9x + 4)(9x - 4)$

12. $16x^2 - 121$
$(4x + 11)(4x - 11)$

13. $25x^2 - 144$
$(5x + 12)(5x - 12)$

14. $16x^2 - 9$
$(4x + 3)(4x - 3)$

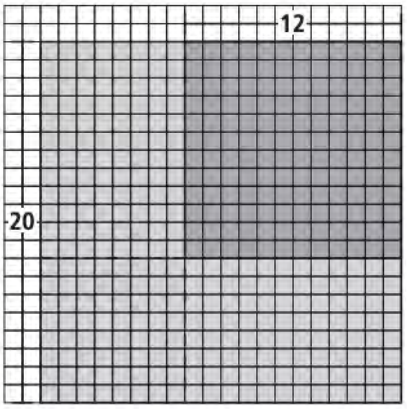
15. $x^2 - 81$
$(x + 9)(x - 9)$

16. $4x^2 - 49$
$(2x + 7)(2x - 7)$

8-7 Activity: Grid-Paper Factoring

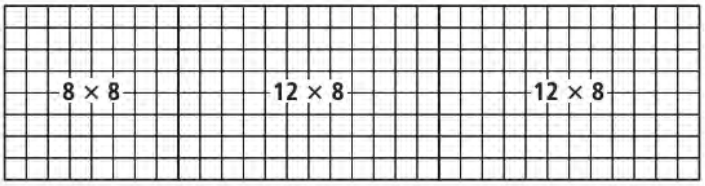
Factoring Special Cases

You can solve factoring problems with paper and scissors.



What is the area of the lightly shaded parts in the grid at the right in terms of 20 and 12?

- Use the graph paper to draw a square 20 units by 20 units and, inside it, a square 12 units by 12 units.
- Cut out the three lightly shaded regions and the darkly shaded region. Put the darkly shaded region aside.
- Tape the three remaining shapes together to form a single rectangle like the one below. Be sure to label your figure.



- Write expressions for the length and width of this rectangle in terms of 20 and 12.

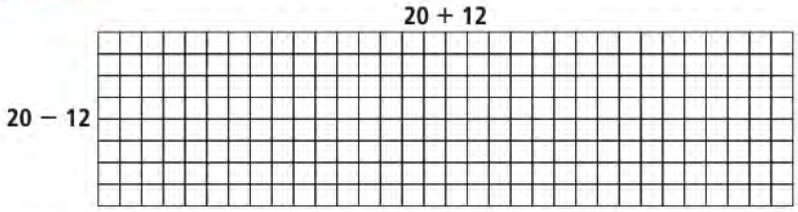
  (20 + 12)(20 − 12)

- Use the formula for the area of a rectangle to find the area of the three combined rectangles.

  256 units^2

- On a separate piece of grid paper (units must be the same size as the one used above), cut out a rectangle whose length is 20 + 12 units and whose width is 20 − 12 units. Use the formula for the area of a rectangle to find the area of this rectangle.

  256 units^2



- How does this expression relate to the expression you wrote for the area of the lightly shaded region?

  Regardless of the rearrangement, the nonshaded region has an area of 256 units^2.

Write $20^2 - 12^2$ as a product of a sum and a difference.

(20 + 12)(20 − 12)

8-7 Think About a Plan

Factoring Special Cases

Interior Design A square rug has an area of $49x^2 - 56x + 16$. A second square rug has an area of $16x^2 + 24x + 9$. What is an expression that represents the difference of the areas of the rugs? Show two different ways to find the solution.

1. What are two methods you could use to solve this problem? subtraction; factoring before subtracting
2. How would you find the difference without factoring? subtract the polynomials
3. What polynomial do you get when you use this method? $33x^2 - 80x + 7$
4. Can you factor that polynomial? yes ; $(11x - 1)(3x - 7)$
5. How could you use factoring to solve the problem? Factoring gives you a second way to find the difference. You can represent the difference in the form $a^2 - b^2$.
6. What do the shape of the rug and the polynomials tell you about how to factor the polynomials for the area of the rugs? The factors of each square polynomial will be the same.
7. Factor each trinomial.

   $49x^2 - 56x + 16 = (\square - \square)(\square - \square) = (\square\square\square)^2$ $(7x - 4)(7x - 4) = (7x - 4)^2$

   $16x^2 + 24x + 9 = (\square + \square)(\square + \square) = (\square\square\square)^2$ $(4x + 3)(4x + 3) = (4x - 3)^2$
8. Use your results from Exercise 7 to write an expression for the difference in the areas.

   $(7x - 4)^2 - (4x + 3)^2$
9. Factor the expression from Exercise 8 using the difference of two squares. Simplify the expressions within each set of parentheses.

   $[(7x - 4) + (4x + 3)][(7x - 4) - (4x + 3)] = (11x - 1)(3x - 7)$
10. Do the two methods give you the same result?

    yes

8-7 Practice

Form G

Factoring Special Cases

Factor each expression.

1. $h^2 + 10h + 25$
   $(h + 5)^2$
2. $v^2 - 14v + 49$
   $(v - 7)^2$
3. $d^2 - 22d + 121$
   $(d - 11)^2$
4. $m^2 + 4m + 4$
   $(m + 2)^2$
5. $q^2 + 6q + 9$
   $(q + 3)^2$
6. $p^2 - 24p + 144$
   $(p - 12)^2$
7. $36x^2 + 60x + 25$
   $(6x + 5)^2$
8. $64x^2 + 48x + 9$
   $(8x + 3)^2$
9. $49n^2 + 14n + 1$
   $(7n + 1)^2$
10. $16s^2 - 72s + 81$
    $(4s - 9)^2$
11. $25r^2 - 80r + 64$
    $(5r - 8)^2$
12. $9g^2 - 24g + 16$
    $(3g - 4)^2$
13. $81w^2 + 144w + 64$
    $(9w + 8)^2$
14. $16e^2 - 88e + 121$
    $(4e - 11)^2$
15. $25j^2 + 100j + 100$
    $(5j + 10)^2$
16. $144f^2 - 24f + 1$
    $(12f - 1)^2$
17. $4a^2 - 36a + 81$
    $(2a - 9)^2$
18. $49d^2 - 84d + 36$
    $(7d - 6)^2$

The given expression represents the area. Find the side length of the square.

19. 
    $64x^2 + 80x + 25$
    $8x + 5$
20. 
    $9y^2 - 24y + 16$
    $3y - 4$
21. 
    $4t^2 + 36t + 81$
    $2t + 9$
22. 
    $36n^2 + 84n + 49$
    $6n + 7$
23. 
    $100w^2 + 20w + 1$
    $10w + 1$
24. 
    $16s^2 + 104s + 169$
    $4s + 13$

25. **Error Analysis** Describe and correct the error made in factoring the expression at the right.

$$175x^2 - 28 = 7(25x^2 - 4)$$
$$= 7(5x - 2)(5x - 2)$$
$$= 7(5x - 2)^2$$

$(25x^2 - 4)$ factors to $(5x - 2)(5x + 2)$, not $(5x - 2)^2$

8-7 Practice (continued)

Form G

Factoring Special Cases

Factor each expression.

26. $m^2 - 49$
$(m + 7)(m - 7)$

27. $c^2 - 100$
$(c + 10)(c - 10)$

28. $p^2 - 16$
$(p + 4)(p - 4)$

29. $4a^2 - 25$
$(2a + 5)(2a - 5)$

30. $64n^2 - 1$
$(8n + 1)(8n - 1)$

31. $25x^2 - 144$
$(5x + 12)(5x - 12)$

32. $50g^2 - 8$
$2(5g + 2)(5g - 2)$

33. $8d^2 - 8$
$8(d + 1)(d - 1)$

34. $27x^2 - 48$
$3(3x + 4)(3x - 4)$

35. $24e^2 - 54$
$6(2e + 3)(2e - 3)$

36. $245k^2 - 20$
$5(7k + 2)(7k - 2)$

37. $112h^2 - 63$
$7(4h + 3)(4h - 3)$

38. $48x^2 + 72x + 27$
$3(4x + 3)^2$

39. $8b^2 + 80b + 200$
$8(b + 5)^2$

40. $48w^2 + 48w + 12$
$12(2w + 1)^2$

41. $45s^2 - 210s + 245$
$5(3s - 7)^2$

42. $45t^2 - 72t + 24$
$3(15t^2 - 24t + 8)$

43. $100z^2 - 120z + 36$
$4(5z - 3)^2$

44. Writing Explain how to recognize a perfect-square trinomial.
The coefficient of the squared term and the constant will be perfect squares. Twice the product of these numbers is the coeffiecient of the middle term.The sign before the constant will be positive.

45. a. Open-Ended Write an expression that shows the factored form of a difference of two squares. Answers may vary. Sample: $(2x + 3)(2x - 3)$

b. Explain how you know that your expression is a difference of two squares.
Answers may vary. Sample: $4x^2 - 9$; $4x^2$ and 9 are squares and they are separated by a subtraction.

Factor each expression.

46. $36s^8 - 60s^4 + 25$
$(6s^4 - 5)^2$

47. $c^{10} - 30c^5d^2 + 225d^4$
$(c^5 - 15d^2)^2$

48. $25n^6 + 40n^3 + 16$
$(5n^3 + 4)^2$

Mental Math **For Exercises 49–51, find a pair of factors for each number by using the difference of two squares.**

49. 24
$24 = 5^2 - 1^2$
$= (5 + 1)(5 - 1) = (6)(4)$

50. 28
$28 = 8^2 - 6^2$
$= (8 - 6)(8 + 6) = (2)(14)$

51. 72
$72 = 9^2 - 3^2$
$= (9 + 3)(9 - 3) = (12)(6)$

52. Reasoning Explain how reversing the rules for multiplying squares of binomials can help you factor a perfect-square trinomial.
When the b term in a trinomial is exactly twice the product of a and c, you can factor it as $(a + b)^2$ or as $(a - b)^2$.

53. Writing The area of a square parking lot is $49p^4 - 84p^2 + 36$. Explain how you would find the length of the parking lot.
Factor $49p^4 - 84p^2 + 36$ to find the length. You get $(7p^2 - 6)^2$ so each side has a length of $(7p^2 - 6)$.

8-7 Practice

Form K

Factoring Special Cases

Factor each expression.

1. $c^2 + 2c + 1$
   $(c + 1)^2$

2. $d^2 - 10d + 25$
   $(d - 5)^2$

3. $p^2 - 24p + 144$
   $(p - 12)^2$

4. $w^2 + 14w + 49$
   $(w + 7)^2$

5. $s^2 + 16s + 64$
   $(s + 8)^2$

6. $9g^2 + 24g + 16$
   $(3g + 4)^2$

7. $25m^2 - 60m + 36$
   $(5m - 6)^2$

8. $4q^2 - 32q + 64$
   $4(q - 4)^2$

9. $49y^2 - 84y + 36$
   $(7y - 6)^2$

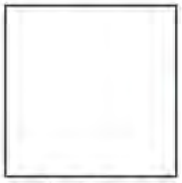
10. $121n^2 - 66n + 9$
    $(11n - 3)^2$

11. $81x^2 - 18x + 1$
    $(9x - 1)^2$

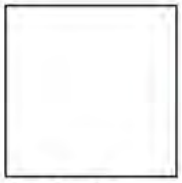
12. $100t^2 - 100t + 25$
    $25(2t - 1)^2$

The given expression represents the area. Find the side length of the square.

13. 
    $36w^2 + 12w + 1$
    $6w + 1$

14. 
    $81w^2 - 72w + 16$
    $9w - 4$

15. 
    $9w^2 - 48w + 64$
    $3w - 8$

16. 
    $121w^2 - 66w + 9$
    $11w - 3$

17. **Writing** How can you tell that $x^2 - 19x + 90$ is not a perfect square trinomial?
    Sample: 90 is not a perfect square.

8-7 Practice (continued)

Form K

Factoring Special Cases

Factor each expression.

18. $b^2 - 121$
$(b + 11)(b - 11)$

19. $d^2 - 81$
$(d + 9)(d - 9)$

20. $f^2 - 625$
$(f + 25)(f - 25)$

21. $108x^2 - 3$
$3(6x + 1)(6x - 1)$

22. $50n^2 - 8$
$2(5n + 2)(5n - 2)$

23. $405z^2 - 245$
$5(9z + 7)(9z - 7)$

24. $216h^2 - 150$
$6(6h + 5)(6h - 5)$

25. $28y^2 - 28$
$28(y + 1)(y - 1)$

26. $50t^2 + 40t + 8$
$2(5t + 2)(5t + 2)$

27. $12n^2 - 36n + 27$
$3(2n - 3)(2n - 3)$

28. $180a^2 - 300a + 125$
$5(6a - 5)(6a - 5)$

29. $250k^2 - 200k + 40$
$10(5k - 2)(5k - 2)$

30. Writing Explain how to recognize a difference of two squares.
The expression is the difference of two terms that are both perfect squares.

31. a. Open-Ended Write an expression that shows the factored form of a perfect-square trinomial.
Answers may vary. Sample: $(5x + 3)(5x + 3)$ or $(5x + 3)^2$

b. Explain how you know your expression is a perfect-square trinomial when expanded. It is in the form $a^2 + 2ab + b^2$.

Mental Math For Exercises 32–34, find a pair of factors for each number by using the difference of two squares.

32. 84 (14)(6)

33. 55 (11)(5)

34. 80 (20)(4)

35. Writing The area of a square painting is $225x^4 + 240x^2 + 64$. Explain how you would find a possible length of one side of the painting.
Since the trinomial is a perfect-square trinomial, the length of the side could be a factor of the trinomial.

8-7 Enrichment

Factoring Special Cases

The surface area of a cube is determined by the formula $SA = 6s^2$, where s is the length of a side of the cube. You can use this formula to analyze a polynomial that represents the surface area of a cube.

Start by dividing the polynomial by 6. This will leave an expression for of the area of one face of the cube. You can see that the area is a perfect-square trinomial. Reverse the rules for multiplying squares of binomials to factor the trinomial.

For example, a cube with a surface area of $24x^2 + 24x + 6$ has a side measure of $2x + 1$.

$$6s^2 = 24x^2 + 24x + 6$$
$$s^2 = \frac{24x^2 + 24x + 6}{6} = 4x^2 + 4x + 1$$
$$s^2 = (2x + 1)(2x + 1)$$
$$s = 2x + 1$$

The surface area of a rectangular prism with two square faces is determined by the formula $SA = 4ls + 2s^2$, where l is the length and s is the measure of the side of the square face. If you are given the surface area and the area of the square face, you can determine the dimensions of the rectangular prism.

Suppose a rectangular prism has a surface area of $24x + 30$ and each square face measures $9\ \text{cm}^2$.

$24x + 30 - 18 = 24x + 12$	Subtract the area of the square faces.
$\frac{24x + 12}{4} = 6x + 3$	Divide by 4 to get the area of each remaining side.
$\frac{6x + 3}{3} = 2x + 1$	Divide by the side length of the square base, or the square root of the base's area.

1. The surface area of a cube is $96x^2 + 144x + 54$. What is the measure of each side?
   $4x + 3$
2. The surface area of a cube is $54x^2 - 36x + 6$. What is the measure of each side?
   $3x - 1$
3. The surface area of a cube is $864x^2 + 720x + 150$. What is the measure of each side?
   $12x + 5$
4. The surface area of a rectangular prism is $100x + 90$. The areas of the two square faces of the prism are $25\ \text{m}^2$ each. What are the dimensions of the rectangular prism?
   5, 5, and $5x + 2$
5. The surface area of a rectangular prism is $2x^2 + 48x + 88$. The areas of the two square faces of the prism are $x^2 + 4x + 4$ each. What are the dimensions of the rectangular prism?
   10, $x + 2$, and $x + 2$

8-8 Additional Vocabulary Support

Factoring by Grouping

Use the list to complete the diagram.

common factors	factor	GCF	pair of binomial factors	squares

Steps for Factoring a Polynomial Completely

1. Factor out the GCF.

2. If the polynomial has two or three terms, look for a difference of two squares, a perfect-square trinomial, or a pair of binomial factors.

3. If the polynomial has four or more terms, group terms and factor to find common binomial factors.

4. Make sure there are no common factors other than 1.

8-8 Reteaching

Factoring by Grouping

You can factor some higher-degree polynomials by grouping terms and factoring out the GCF to find the common binomial factor. Make sure to factor out a common GCF from all terms first before grouping.

Problem

What is the factored form of $2b^4 - 8b^3 + 10b^2 - 40b$?

$2b^4 - 8b^3 + 10b^2 - 40b = 2b(b^3 - 4b^2 + 5b - 20)$	$2b$ is the GCF of all four terms. Factor out $2b$ from each term.
$= 2b[b^2(b - 4) + 5(b - 4)]$	Group terms into pairs and look for the GCF of each pair. b^2 is the GCF of the first pair, and 5 is the GCF of the second pair.
$= 2b(b^2 + 5)(b - 4)$	$b - 4$ is the common binomial factor. Use the Distributive Property to rewrite the expression.

Multiply to check your answer.

$2b(b^2 + 5)(b - 4) = 2b(b^3 + 5b - 4b^2 - 20)$	Multiply $b^2 + 5$ and $b - 4$.
$= 2b^4 + 10b^2 - 8b^3 - 40b$	Multiply by $2b$.
$= 2b^4 - 8b^3 + 10b^2 - 40b$ ✓	Reorder the terms by degree.

The factored form of $2b^4 - 8b^3 + 10b^2 - 40b$ is $2b(b^2 + 5)(b - 4)$.

Exercises

Factor completely. Show your steps.

1. $4x^4 + 8x^3 + 12x^2 + 24x$
   $4x(x^2 + 3)(x + 2)$
2. $24y^4 + 6y^3 + 36y^2 + 9y$
   $3y(2y^2 + 3)(4y + 1)$
3. $72z^4 + 48z^3 + 126z^2 + 84z$
   $6z(4z^2 + 7)(3z + 2)$
4. $2e^4 - 8e^3 + 18e^2 - 72e$
   $2e(e^2 + 9)(e - 4)$
5. $12f^3 - 36f^2 + 60f - 180$
   $12(f^2 + 5)(f - 3)$
6. $16g^4 - 56g^3 + 64g^2 - 224g$
   $8g(g^2 + 4)(2g - 7)$
7. $56m^3 - 28m^2 - 42m + 21$
   $7(4m^2 - 3)(2m - 1)$
8. $40n^4 - 60n^3 - 50n^2 + 75n$
   $5n(4n^2 - 5)(2n - 3)$
9. $60x^3 - 90x^2 - 30x + 45$
   $15(2x^2 - 1)(2x - 3)$
10. $12p^5 + 8p^4 + 18p^3 + 12p^2$
    $2p^2(2p^2 + 3)(3p + 2)$
11. $6r^3 + 9r^2 - 60r$
    $3r(2r - 5)(r + 4)$
12. $20s^6 - 50s^5 - 30s^4$
    $10s^4(2s + 1)(s - 3)$

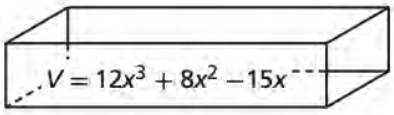
8-8 Reteaching (continued)

Factoring by Grouping

Polynomials can be used to express the volume of a rectangular prism. They can sometimes be factored into 3 expressions to represent possible dimensions of the prism. The three factors are the length, width, and height.

Problem

The plastic storage container to the right has a volume of $12x^3 + 8x^2 - 15x$. What linear expressions could represent possible dimensions of the storage container?



$12x^3 + 8x^2 - 15x = x(12x^2 + 8x - 15)$	Factor out x, the GCF for all three terms.
$= x(12x^2 + 18x - 10x - 15)$	ac is -180 and b is 8. Break $8x$ into two terms that have a sum of $8x$ and a product of $-180x^2$.
$= x[6x(2x + 3) - 5(2x + 3)]$	Group the terms into pairs and factor out the GCF from each pair. The GCF of the first pair is $6x$. The GCF of the second pair is -5.
$= x(6x - 5)(2x + 3)$	$2x + 3$ is the common binomial term. Use the Distributive Property to reorganize the factors.

Multiply to check your answer.

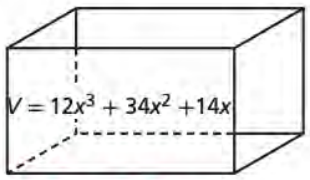
$x(6x - 5)(2x + 3) = x(12x^2 + 18x - 10x - 15)$	Multiply $6x - 5$ and $2x + 3$.
$= x(12x^2 + 8x - 15)$	Combine like terms.
$= 12x^3 + 8x^2 - 15x$ ✓	Multiply by x.

Possible dimensions of the storage container are x, $6x - 5$, and $2x + 3$.

Exercises

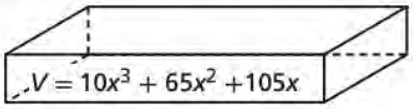
Find linear expressions for the possible dimensions of each rectangular prism.

13.



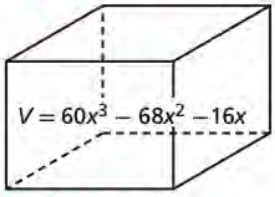
$2x$, $3x + 7$, $2x + 1$

14.



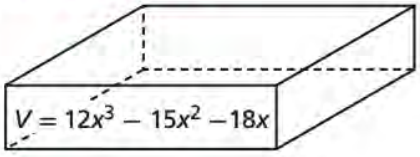
$5x$, $2x + 7$, $x + 3$

15.



$4x$, $5x + 1$, $3x - 4$

16.



$3x$, $4x + 3$, $x - 2$

8-8 Think About a Plan

Factoring by Grouping

Art The pedestal of a sculpture is a rectangular prism with a volume of $63x^3 - 28x$. What expressions can represent the dimensions of the pedestal? Use factoring.

KNOW

1. The pedestal of the sculpture is shaped like a rectangular prism.

2. The volume of the pedestal is $63x^3 - 28x$.

3. The formula you can use to find the dimensions of the pedestal is $V = lwh$.

NEED

4. To solve the problem you need to find 3 factors

PLAN

5. Factor out the GCF from the volume of the pedestal. $7x(9x^2 - 4)$

6. What type of expression is of the remaining expression? difference of two squares

7. Factor the expression completely. $7x(3x - 2)(3x + 2)$

8. What expressions represent possible dimensions of the pedestal?

   $7x$, $(3x - 2)$, and $(3x + 2)$

8-8 Practice

Form G

Factoring by Grouping

Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

1. $12x^3 + 3x^2 + 20x + 5$

   $3x^2, 5$

2. $6v^3 + 42v^2 + 5v + 35$

   $6v^2, 5$

3. $8t^3 + 36t^2 + 2t + 9$

   $4t^2, 1$

4. $10s^3 + 35s^2 + 6s + 21$

   $5s^2, 3$

5. $9m^3 - 6m^2 + 12m - 8$

   $3m^2, 4$

6. $8w^3 + 6w^2 - 28w - 21$

   $2w^2, -7$

7. $7r^3 + 16r^2 - 9r - 72$

   $r^2, -9$

8. $21x^3 - 28x^2 - 6x + 8$

   $7x^2, -2$

Factor each expression.

9. $8j^3 + 4j^2 + 10j + 5$

   $(4j^2 + 5)(2j + 1)$

10. $2m^3 + 8m^2 + 9m + 36$

    $(2m^2 + 9)(m + 4)$

11. $10s^3 + 25s^2 + 8s + 20$

    $(5s^2 + 4)(2s + 5)$

12. $6x^3 + 9x^2 + 2x + 3$

    $(3x^2 + 1)(2x + 3)$

13. $21x^3 + 6x^2 - 28x - 8$

    $(3x^2 - 4)(7x + 2)$

14. $8w^3 + 12w^2 + 10w + 15$

    $(4w^2 + 5)(2w + 3)$

15. $18r^3 - 12r^2 + 21r - 14$

    $(6r^2 + 7)(3r - 2)$

16. $36n^3 - 27n^2 - 8n + 6$

    $(9n^2 - 2)(4n - 3)$

17. $110b^3 + 77b^2 - 60b - 42$

    $(11b^2 - 6)(10b + 7)$

18. $64d^3 - 40d^2 - 24d + 15$

    $(8d^2 - 3)(8d - 5)$

19. $10s^3 + 80s^2 - 7s - 56$

    $(10s^2 - 7)(s + 8)$

20. $25j^3 + 15j^2 - 5j - 3$

    $(5j^2 - 1)(5j + 3)$

21. $24c^3 - 84c^2 + 10c - 35$

    $(12c^2 + 5)(2c - 7)$

22. $27f^3 + 9f^2 - 24f - 8$

    $(9f^2 - 8)(3f + 1)$

8-8 Practice (continued)

Form G

Factoring by Grouping

Factor completely.

23. $32x^3 + 8x^2 + 48x + 12$

$4(2x^2 + 3)(4x + 1)$

24. $45w^4 - 36w^3 + 15w^2 - 12w$

$3w(3w^2 + 1)(5w - 4)$

25. $32k^4 - 16k^3 + 12k^2 - 6k$

$2k(8k^2 + 3)(2k - 1)$

26. $6g^3 + 18g^2 + 60g + 180$

$6(g^2 + 10)(g + 3)$

27. $30b^4 - 45b^3 - 10b^2 + 15b$

$5b(3b^2 - 1)(2b - 3)$

28. $32m^3 + 72m^2 - 80m - 180$

$4(2m^2 - 5)(4m + 9)$

29. $63j^4 + 84j^3 - 18j^2 - 24j$

$3j(7j^2 - 2)(3j + 4)$

30. $96n^3 - 240n^2 - 168n + 420$

$12(4n^2 - 7)(2n - 5)$

31. $12e^4 + 18e^3 + 36e^2 + 54e$

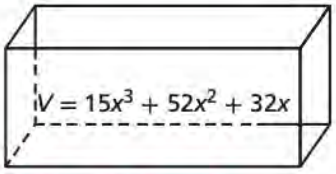
$6e(e^2 + 3)(2e + 3)$

32. $60a^5 - 72a^4 - 210a^3 + 252a^2$

$6a^2(2a^2 - 7)(5a - 6)$

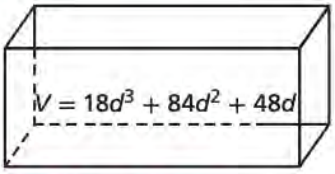
Find linear expressions for the possible dimensions of each rectangular prism.

33.



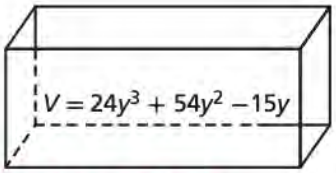
$x, 5x + 4, 3x + 8$

34.



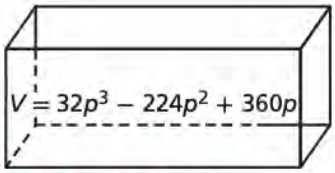
$6d, 3d + 2, d + 4$

35.



$3y, 4y - 1, 2y + 5$

36.



$8p, 2p - 5, 2p - 9$

37. A shipping box in the shape of a rectangular prism has a volume of $12x^3 + 32x^2 + 20x$. What linear expressions can represent possible dimensions of the box?

$4x, 3x + 5, x + 1$

38. Error Analysis Describe and correct the error made in factoring completely.

$$\cancel{16x^4 + 24x^3 + 64x^2 + 96x = 4x(4x^3 + 6x^2 + 16x + 24)}$$
$$\cancel{= 4x[2x^2(2x + 3) + 8(2x + 3)]}$$
$$\cancel{= 4x(2x^2 + 8)(2x + 3)}$$

In the first step, the GCF is $8x$, not $4x$.

39. Open-Ended Write a 3-term expression for the volume of a rectangular prism that you can factor by grouping. Factor your polynomial.

Answers may vary. Sample: $x^5 + 4x^4 + 3x^3 = x^3(x + 3)(x + 1)$

8-8 Practice

Form K

Factoring by Grouping

Find the GCF of the first two terms and the GCF of the last two terms for each polynomial.

1. $6n^3 + 3n^2 + 10n + 5$ $3n^2$; 5

2. $12z^3 + 36z^2 + 4z + 12$ $12z^2$; 4

3. $9k^3 + 45k^2 + 2k + 10$ $9k^2$; 2

4. $11a^3 + 33a^2 + 8a + 24$ $11a^2$; 8

5. $2f^3 + 5f^2 - 4f - 10$ f^2; -2

6. $16d^3 - 24d^2 - 6d + 9$ $8d^2$; -3

Factor each expression.

7. $6x^3 - 4x^2 + 15x - 10$
   $(2x^2 + 5)(3x - 2)$

8. $5q^3 - 40q^2 - 4q + 32$
   $(5q^2 - 4)(q - 8)$

9. $28m^3 + 7m^2 - 8m - 2$
   $(7m^2 - 2)(4m + 1)$

10. $3p^3 + 5p^2 + 9p + 15$
    $(p^2 + 3)(3p + 5)$

11. $18y^3 - 6y^2 - 63y + 21$
    $3(2y^2 - 7)(3y - 1)$

12. $3t^3 - 18t^2 + 5t - 30$
    $(3t^2 + 5)(t - 6)$

13. $250c^3 - 250c^2 + 100c - 100$
    $50(5c^2 + 2)(c - 1)$

14. $18g^3 - 33g^2 + 30g - 55$
    $(3g^2 + 5)(6g - 11)$

15. $88n^3 + 77n^2 - 72n - 63$
    $(11n^2 - 9)(8n + 7)$

16. $50h^3 - 40h^2 + 60h - 48$
    $2(5h^2 + 6)(5h - 4)$

17. $24b^3 - 96b^2 - 14b + 56$
    $2(12b^2 - 7)(b - 4)$

18. $54r^3 + 9r^2 - 6r - 1$
    $(9r^2 - 1)(6r + 1)$

8-8 Practice (continued)

Form K

Factoring by Grouping

Factor completely.

19. $49s^3 + 14s^2 + 14s + 4$
$(7s^2 + 2)(7s + 2)$

20. $32h^4 + 72h^3 + 36h^2 + 81h$
$h(8h^2 + 9)(4h + 9)$

21. $42z^4 - 48z^3 - 7z^2 + 8z$
$z(6z^2 - 1)(7z - 8)$

22. $60p^3 + 48p^2 + 25p + 20$
$(12p^2 + 5)(5p + 4)$

23. $26n^4 - 14n^3 + 91n^2 - 49n$
$n(2n^2 + 7)(13n - 7)$

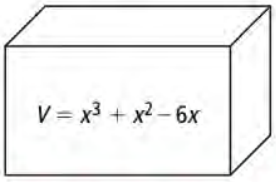
24. $40t^3 + 28t^2 - 30t - 21$
$(4t^2 - 3)(10t + 7)$

25. $45k^4 - 9k^3 + 10k^2 - 2k$
$k(9k^2 + 2)(5k - 1)$

26. $18b^5 - 3b^4 + 30b^3 - 5b^2$
$b^2(3b^2 + 5)(6b - 1)$

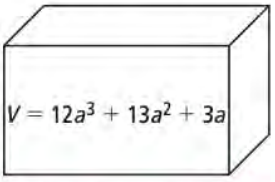
Find linear expressions for the possible dimensions of each rectangular prism.

27.



x by $(x + 3)$ by $(x - 2)$

28.



a by $(3a + 1)$ by $(4a + 3)$

29. A storage bin in the shape of a rectangular prism has a volume of $10x^3 + 9x^2 + 2x$. What linear expressions can represent possible dimensions of the bin? x; $(5x + 2)$; $(2x + 1)$

30. Writing Describe the first step to look for in factoring a cubic expression containing four terms.
Check to see if you can factor a GCF from all four terms.

31. Open-Ended Write a 4-term expression that you can factor by grouping. Factor your polynomial.
Answers may vary. Sample: $4x^3 + 36x^2 + 7x + 63 = (4x^2 + 7)(x + 9)$

8-8 Enrichment

Factoring by Grouping

Pascal's triangle is named after French mathematician Blaise Pascal, but this special number pattern had been studied in India, China, Persia, and Italy long before Pascal. To generate Pascal's triangle, start with the number 1 in Row 0. Each successive row has a 1 at both ends. Add the numbers directly above-left and above-right to find the new value.

Row 0	1
Row 1	1 1
Row 2	1 2 1
Row 3	1 3 3 1
Row 4	1 4 6 4 1
Row 5	1 5 10 10 5 1
Row 6	1 6 15 20 15 6 1
Row 7	1 7 21 35 35 21 7 1
Row 8	1 8 28 56 70 56 28 8 1

You can use Pascal's triangle to quickly expand a binomial expression. The exponent tells you the row number to choose. The numbers in the correct row are the coefficients to use in the expansion.

To expand $(a + b)^4$, look to Row 4. The coefficients are 1, 4, 6, 4, 1. Expand the variables, raising the first variable to 4 and decreasing by one for each term. Raise the second variable to 0 and increase by 1 at each new term. Multiply each term by the coefficients: $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

You can also use Pascal's triangle to factor polynomials that are expansions of binomial expressions. Arrange the polynomial in standard form. Check to see if the coefficients correspond to a row in Pascal's triangle. Work backwards to factor.

To factor $15xy^2 - y^3 + 125x^3 - 75x^2y$, first rearrange the terms in standard form: $125x^3 - 75x^2y + 15xy^2 - y^3$. In expansions of binomial expressions the x-exponents decrease by one in every term and y-exponents increase by one. Since the first and last terms have exponents of 3, the binomial is raised to the third power. Find the cube root of 125 to find the coefficient of x: 5. The final term is negative and has a coefficient of one, so the expression is $(5x - y)^3$. Expand the binomial to check your answer:

$$(5x - y)^3 = 1(5x)^3 + 3(5x)^2(-y) + 3(5x)(-y)^2 + 1(-y)^3 = 125x^3 - 75x^2y + 15xy^2 - y^3$$

Expand the binomial using Pascal's triangle.

1. $(4k + j)^4$ $256k^4 + 256k^3j + 96k^2j^2 + 16kj^3 + j^4$

2. $(7x - y)^7$
$823{,}543x^7 - 823{,}543x^6y + 352{,}947x^5y^2 - 84{,}035x^4y^3 + 12{,}005x^3y^4 - 1029x^2y^5 + 49xy^6 - y^7$

Factor the polynomial using Pascal's triangle. Then expand the binomial to check your answer.

3. $8a^3 + 12a^2b + 6ab^2 + b^3$ $(2a + b)^3$

4. $40x^2y^3 + 32x^5 + 10xy^4 + 80x^4y + y^5 + 80x^3y^2$ $(2x + y)^5$

5. $1215x^4y^2 + 135x^2y^4 + 729x^6 + 18xy^5 + y^6 + 1458x^5y + 540x^3y^3$ $(3x + y)^6$