THE SCHOOL DISTRICT OF PHILADELPHIA

Geometry Learning Guide

Grade: HS **Subject:** Geometry (Worksheets taken from Pearson)

THE SCHOOL DISTRICT OF PHILADELPHIA

Topic: Relationships Within Triangles

What Your Student is Learning: Students will coordinate geometry to find relationships within triangles. They will use the Midpoint Formula to find midsegments of triangles and use the Distance Formula to examine relationships in triangles. Students will also examine inequalities in up to two triangles. They will write indirect proofs.

Background and Context for Parents: This unit builds on your students' previous understandings and skills related to angles and triangles.

- A coordinate system in a plane is formed by two perpendicular number lines, called the x- and y-axes, and the quadrants they form. It is possible to verify some complex truths using deductive reasoning in combination with Distance, Midpoint and Slope formulas.

- Definitions establish meanings and remove possible misunderstanding. Other truths are more complex and difficult to see. It is often possible to verify complex truths by reasoning from simpler ones by using deductive reasoning.

- Some attributes of geometric figures, such as length, area, volume, and angle measure, are measurable. Units are used to describe these attributes.

Indirect Proof

Indirect proof involves assuming the opposite of a statement you wish to prove, then showing a contradiction of the given information. Indirect proof is usually used in situations where there are only two possibilities: the given is true or the given is false. If there are more possibilities, each must be contradicted in order to use an indirect proof.

For example:

Given: Two distinct lines in a plane do not intersect.

Prove: The two lines are parallel.

Two distinct lines in a plane are either parallel, or they are not parallel. Since only two possibilites exist, an indirect proof can be used.

1. Assume the opposite of what you want to prove:
   The two lines are *not* parallel.
2. Show that this assumption leads to a contradiction:
   If they are *not* parallel, then they intersect.
   This contradicts the given information that they do not intersect.
3. Conclude the assumption is false.
   It is false that the two lines are not parallel.
   There is only one other possibility: the lines are parallel.

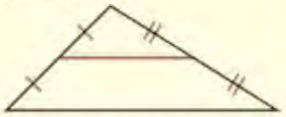
Common Errors With Indirect Proof

Sometimes it is not safe to assume that there are only two possibilities. To demonstrate this, consider the example above. Notice that it was given that the two lines were in a plane. If the two lines were in three-dimensional space, there would be a third possibility: the two lines might be skew. In this case, to prove that the two lines are parallel by indirect proof, students would also need to have sufficient given information to show that the lines being skew leads to a contradiction, otherwise they could not prove the lines parallel from the given information.

Special Segments in Triangles

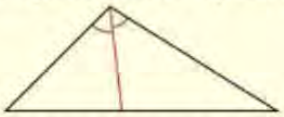
Five special segments are defined in this chapter.

Midsegment is a segment that connects the midpoints of two sides.

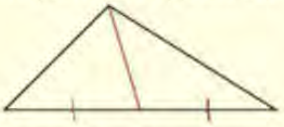


Perpendicular Bisector is a segment that bisects a side and is perpendicular to that side.

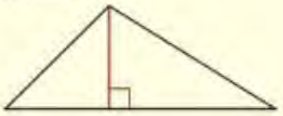
Angle Bisector is a segment that bisects an angle.



Median is a segment that connects a vertex to the midpoint of the opposite side.



Altitude is a segment from a vertex, perpendicular to the opposite side.



Common Errors With Special Segments in Triangles

Special segments can often be difficult to distinguish from each other. Students should use angle and segment congruency marks and right angle marks (as above) to help make clear the purpose of a given segment.

Ways to support your student:

- Read the problem outloud to them.

- Before giving your student the answer to their question or specific help, ask them "What have you tried so far?, What do you know?, What might be a next step?
- After your student has solved it, and before you tell them it's correct or not, have them explain to you how they got their solution and if they think their answer makes sense.
- Sometimes students have trouble accepting the fact that any three given sides may not form a triangle. Giving them experiences in trying to form triangles using pieces of spaghetti before studying the inequality theorems will help them go from the concrete to the abstract.
- When checking to see if three segments can be combined to form a triangle, students must make sure they are comparing the sums of the lengths of the two shorter sides to the length of the longest side.

Online Resources for Students:
Desmos- **https://tinyurl.com/yac8mmqn** **-** Investigating congruent triangles
Khan Academy- Solving Similar Triangles
https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-solving-similar-triangles/v/similarity-example-problems
Geogebra Applet- https://www.geogebra.org/m/dVUczkUm#material/nEqHyhVN

Learning Support for Mathematics

For students that are approaching grade level and have learning gaps/ differences in mathematics, provide numerous opportunities for explorations at the concrete (manipulatives) and representational (visual) levels before progressing to the abstract (numbers) level. Students that need learning supports should be provided with:

- Intensive Direct Instruction and daily guided practice
- scaffolded supports
- the use of visuals as models and aids
- numerous opportunities to think out loud
- support to help them understand the why
- use of manipulatives and tools to support understanding
- Bar Modeling Representations to decode word problems
- the use of mnemonics to enhance retention of skills
- daily practice with basic facts
- the presentation of content in varied contexts and varied levels
- opportunities to use diagrams and draw math concepts
- graph paper to support understanding
- numerous opportunities to draw pictures of word problems
- the use of smaller numbers to address number operations
- opportunities for success to build a growth mindset
- computer time to allow for needed practice
- opportunities to engage in metacognition (the building and reinforcing of thinking and reasoning) skills

(Scaffolded supports means to introduce the skill one step at a time – allowing the student to understand one section part, before moving on to
the next part) ex. 5+ 1=6, 9+1=10, 24+1=25- it is the same as "what
number comes after 5, after 9, after 24
https://youtu.be/5hWDbSx_kdo

- **Visuals as models and aides**

(Pictures of objects that can be used to help students understand the math)
https://studentsatthecenterhub.org/resource/helping-struggling-students-build-a-growth-mindset/

- **Thinking out loud**

(Allows students to talk and think about the skills they are learning, which allows them to better remember the skill)
https://youtu.be/f-4N7OxSMok

- **Understanding the why**

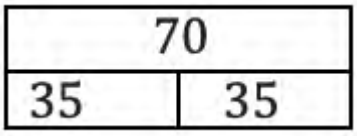
(When students understand why a strategy works, they will apply it to other skills) ex. 5x = 5, 45x1= 45, 320x1=320

- **Manipulatives and Tools**

(Manipulatives can be counters, beans, blocks, etc. – Tools can be rulers, calculators, scales, etc.) **https://youtu.be/uWBZF-Lyq58**

- **Bar Modeling Representations**

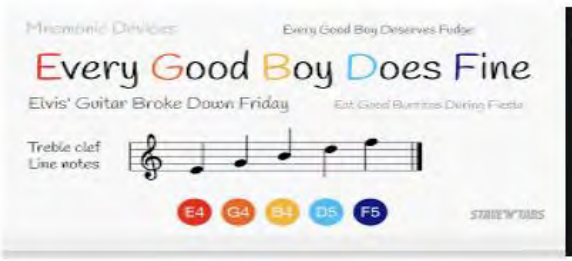
(Bar Modeling Representations consist of visuals that help students under the skill they are learning. Ex.



https://youtu.be/TbayTZvS_bc

- **Mnemonics**

(Mnemonics consist of strategies to help students remember skills – ex.



https://youtu.be/dXvvGc9TldY

- **Basic Facts**

(Basic facts include addition, subtraction, division, multiplication facts –
ex. 8+2=10, 2+8=10, 10-2=8, 10-8=2 / 2x5=10, 5x2=10, 10/2=5, 10/5=2
https://youtu.be/TbayTZvS_bc

- **Content with varied contexts and varied levels**

Means to show student how to solve a problem different ways to allow them to use the skill that way they understand best
https://youtu.be/FVg9n0l0Gf0

- **Diagrams**

(Diagrams provide students with visuals / pictures that help them solve the problem and they help them read the problem with less words)
https://youtu.be/TbayTZvS_bc

- **Graph paper**

(Graph paper helps students to solve the problem by making it visual / easier to see the answer)
https://youtu.be/mX43cn3IASI

- **Drawing Pictures**

(Drawing pictures allow students to show they can solve the problem without using words that they may not know or be able to write)
https://youtu.be/TbayTZvS_bc

- **Smaller Numbers**

(The use of smaller numbers can helps students understand the process of a skill, so that when they move on to bigger numbers, they will see that the process is still the same, they acquire understanding of the skill) ex. 5x = 5, 45x1= 45, 320x1=320

- **Growth Mindset**

(A growth mindset is a process that helps to improve intelligence (thinking), ability (skill) and performance (actions). This means that by helping students to develop a growth mindset, we can help them to learn to think and be problem solvers. This is a process that occurs over time by helping them improve by building success over time.
https://studentsatthecenterhub.org/resource/helping-struggling-students-build-a-growth-mindset/

- **Computer Time**

(Computer time allows students to use websites, games, activities that will help them learn math skills and concepts)
mathgametime.com, pbs.com, bestkidsolutions.com, firstinmath.com, helpingkidsrise.org

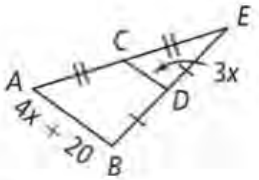
- **Metacognition**

(Metacognition means to help students think about what they are thinking, the steps they are using, the words and numbers that they are using- It helps students to better focus on the skills they are using- it is a process that occurs over time) / **https://youtu.be/HKFOhd5sMEc/**
http://www.spencerauthor.com/metacognition/

5-1 Additional Vocabulary Support

Midsegments of Triangles

There are two sets of note cards below that show how to find *AB* and *CD* for the triangle at the right. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.



Think Cards

Use x to find AB and CD.

Subtract 2x from each side.

Simplify the right side of the equation.

Substitute expressions for the length of each side.

Identify $\overline{CD}$ as the midsegment of $\triangle AEB$. Use the Triangle Midsegment Theorem.

Write Cards

$CD = \frac{1}{2}AB$

$3x - 2x = 2x + 10 - 2x$

$x = 10$

$3x = 2x + 10$

$AB = 4(10) + 20 = 60$

$CD = 3(10) = 30$

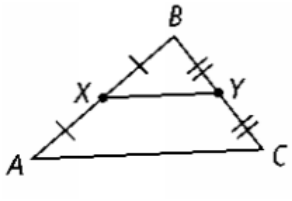
$3x = \frac{1}{2}(4x + 20)$

Think	Write
First, you should	**Step 1**
Second, you should	**Step 2**
Third, you should	**Step 3**
Next, you should	**Step 4**
Finally, you should	**Step 5**

5-1 Reteaching

Midsegments of Triangles

Connecting the midpoints of two sides of a triangle creates a segment called a *midsegment* of the triangle.



Point X is the midpoint of $\overline{AB}$.

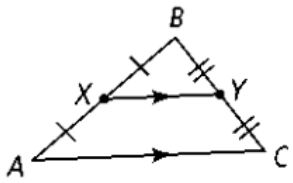
Point Y is the midpoint of $\overline{BC}$.

So, $\overline{XY}$ is a midsegment of ΔABC.

There is a special relationship between a midsegment and the side of the triangle that is not connected to the midsegment.

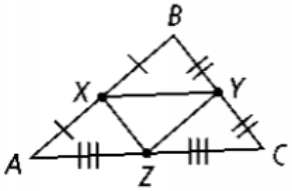
Triangle Midsegment Theorem

- The midsegment is parallel to the third side of the triangle.
- The length of the midsegment is half the length of the third side.



$\overline{XY} \parallel \overline{AC}$ and $XY = \frac{1}{2}AC$.

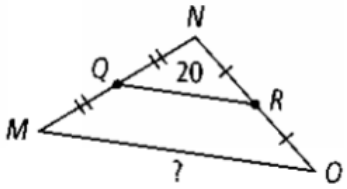
Connecting each pair of midpoints, you can see that a triangle has three midsegments.



$\overline{XY}$, $\overline{YZ}$, and $\overline{ZX}$ are all midsegments of ΔABC.

Because Z is the midpoint of $\overline{AC}$, $XY = AZ = ZC = \frac{1}{2}AC$.

Problem



$\overline{QR}$ is a midsegment of ΔMNO.

What is the length of $\overline{MO}$?

Start by writing an equation using the Triangle Midsegment Theorem.

$$\frac{1}{2}MO = QR$$
$$MO = 2QR$$
$$= 2(20)$$
$$= 40$$

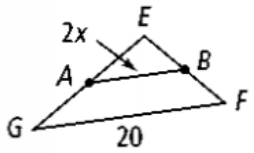
So, $MO = 40$.

5-1 Reteaching (continued)

Midsegments of Triangles

Problem

$\overline{AB}$ is a midsegment of ΔGEF. What is the value of x?



$2AB = GF$

$2(2x) = 20$

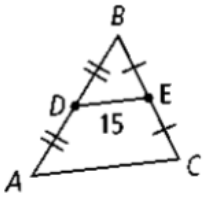
$4x = 20$

$x = 5$

Exercises

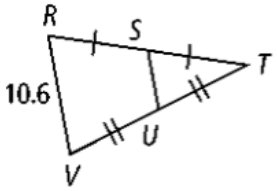
Find the length of the indicated segment.

1. AC

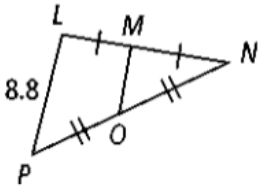


2. TU

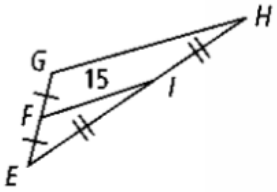
3. SU



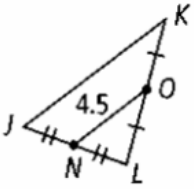
4. MO



5. GH



6. JK



Algebra In each triangle, $\overline{AB}$ is a midsegment. Find the value of x.

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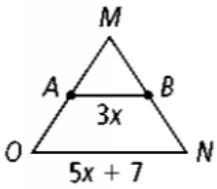

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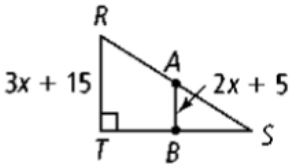

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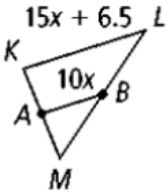

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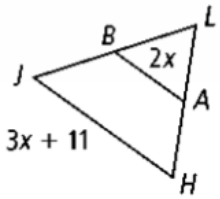

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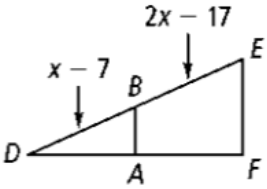

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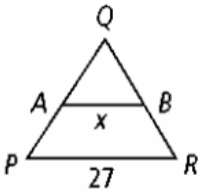

5-1 Think About a Plan

Midsegments of Triangles

Coordinate Geometry The coordinates of the vertices of a triangle are $E(1, 2)$, $F(5, 6)$, and $G(3, 2)$.

a. Find the coordinates of H, the midpoint of $\overline{EG}$, and J, the midpoint of $\overline{FG}$.

b. Show that $\overline{HJ} \parallel \overline{EF}$.

c. Show that $HJ = \frac{1}{2}EF$.

1. In part (a), what formula would you use to find the midpoints of $\overline{EG}$ and $\overline{FG}$? Write this formula.

2. Substitute the x- and y-coordinates of E and G into the formula.

3. Solve to find the coordinates of H, the midpoint of $\overline{EG}$.

4. Use the coordinates of F and G to find the coordinates of J, the midpoint of $\overline{FG}$.

5. In part (b), what information do you need to show $\overline{HJ} \parallel \overline{EF}$? Write the formula you would use.

6. Substitute the x- and y-coordinates of H and J into the formula.

7. Solve to find the slope of $\overline{HJ}$

8. Use the coordinates of E and F to find the slope of $\overline{EF}$.

9. Is $\overline{HJ} \parallel \overline{EF}$? Explain.

10. In part (c), what formula would you use to find HJ and EF? Write this formula.

11. Substitute the x- and y-coordinates of H and J into the formula.

12. Solve to find HJ. Keep in simplest radical form.

13. Use the coordinates of E and F to find EF. Keep in simplest radical form.

14. What is the relationship between HJ and EF?

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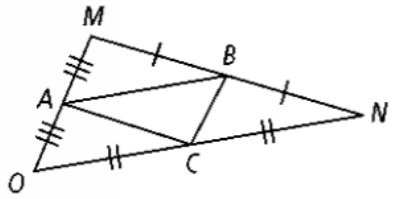
5-1 Practice

Form G

Midsegments of Triangles

Identify three pairs of triangle sides in each diagram.

1.



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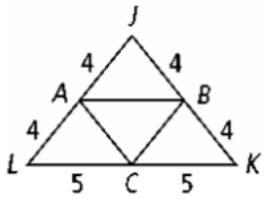


Name the triangle sides that are parallel to the given side.

3. $\overline{AB}$

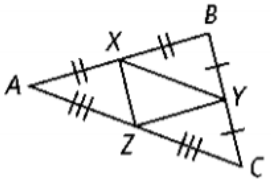
4. $\overline{AC}$

5. $\overline{CB}$

6. $\overline{XY}$

7. $\overline{XY}$

8. $\overline{ZY}$



Points *M*, *N*, and *P* are the midpoints of the sides of □*QRS*. *QR* = 30, *RS* = 30, and *SQ* = 18.

9. Find *MN*.

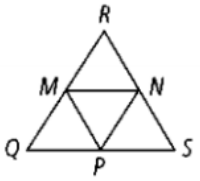
10. Find *MQ*.

11. Find *MP*.

12. Find *PS*.

13. Find *PN*.

14. Find *RN*.



Algebra Find the value of *x*.

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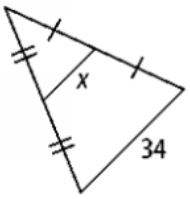

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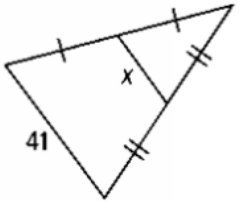

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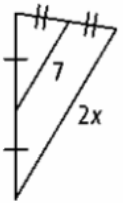

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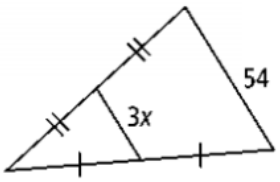

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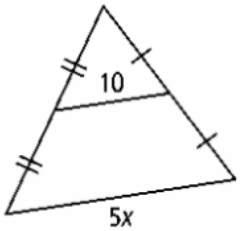
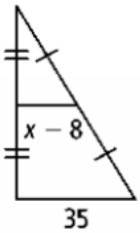


20.



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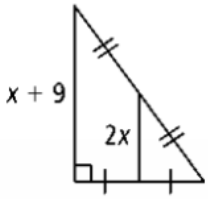

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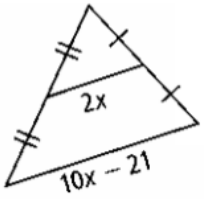

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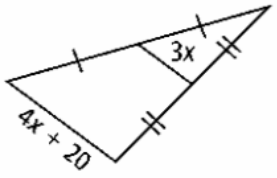
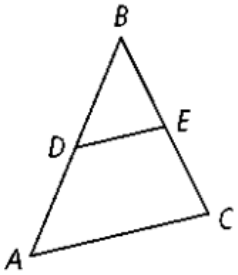


5-1 Practice (continued)

Form G

Midsegments of Triangles

D is the midpoint of $\overline{AB}$. E is the midpoint of $\overline{CB}$.



24. If $m\angle A = 70$, find $m\angle BDE$.

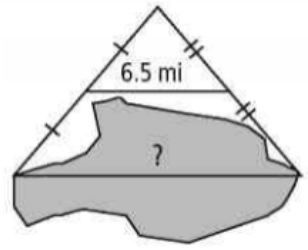
25. If $m\angle BED = 73$, find $m\angle C$.

26. If $DE = 23$, find AC.

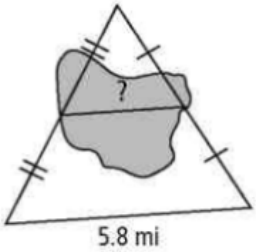
27. If $AC = 83$, find DE.

Find the distance across the lake in each diagram.

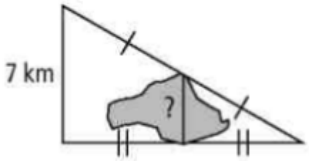
28.



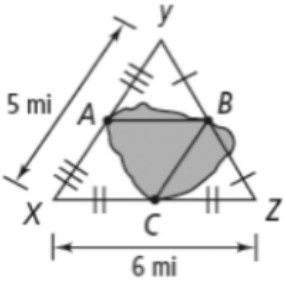
29.



30.



Use the diagram at the right for Exercises 31 and 32.



31. Which segment is shorter for kayaking across the lake, $\overline{AB}$ or $\overline{BC}$? Explain.

32. Which distance is shorter, kayaking from A to B to C, or walking from A to X to C? Explain.

33. Open-Ended Draw a triangle and all of its midsegments. Make a co
what appears to be true about the four triangles that result. What postulates could be used to prove the conjecture?

34. Coordinate Geometry The coordinates of the vertices of a triangle are $K(2, 3)$, $L(2, 1)$, and $M(5, 1)$.

a. Find the coordinates of N, the midpoint of $\overline{KM}$, and P, the midpoint of $\overline{LM}$.

b. Show that $\overline{NP} \parallel \overline{KL}$.

c. Show that $NP = \frac{1}{2}KL$

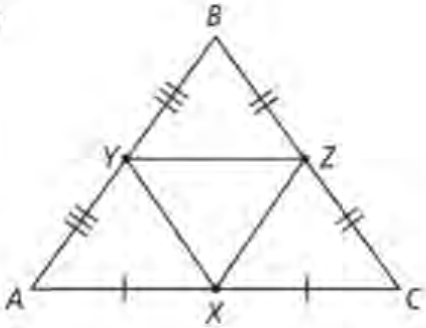
THE SCHOOL DISTRICT OF PHILADELPHIA

5-1 Practice

Form K

Midsegments of Triangles

Identify three pairs of parallel segments in the diagram.

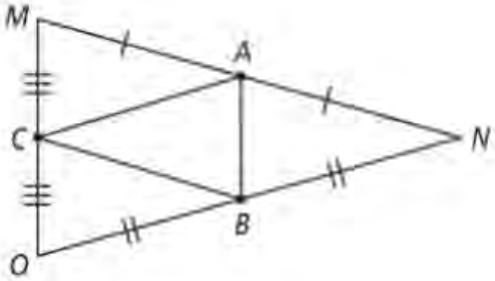


1. $\overline{AB} \parallel$?

2. $\overline{BC} \parallel$?

3. $\overline{AC} \parallel$?

Name the segment that is parallel to the given segment.



4. $\overline{MN}$

5. $\overline{ON}$

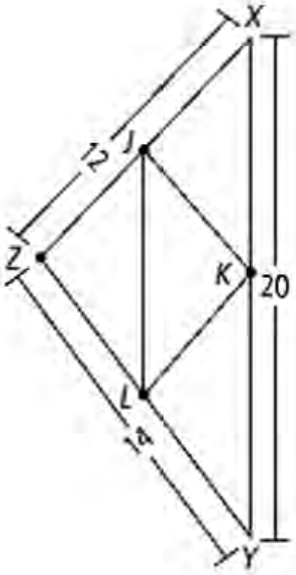
6. $\overline{AB}$

7. $\overline{CB}$

8. $\overline{OM}$

9. $\overline{AC}$

Points *J*, *K*, and *L* are the midpoints of the sides of ΔXYZ.



10. Find LK.

To start, identify what kind of segment $\overline{LK}$ is. Then identify which relationship in the Triangle Midsegment Theorem will help you find the length.

$\overline{LK}$ is a midsegment of $\boxed{}$.

$\overline{LK}$ is parallel to $\boxed{}$.

11. Find YK.

12. Find JK.

13. Find XK.

14. Find JL.

15. Find YL.

16. Find KL.

17. Draw a triangle and label it ABC. Draw all the midpoints and label them. Identify pairs of

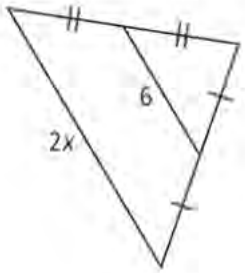
5-1 Practice (continued)

Form K

Midsegments of Triangles

Algebra Find the value of *x*.

18.

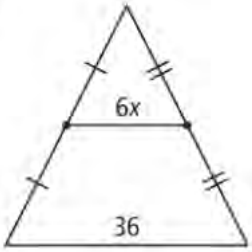


To start, identify the midsegment. Then write an equation to show that its length is half the length of its parallel segment.

The segment with length ☐ is the midsegment.

☐ $= \frac{1}{2} \cdot$ ☐

19.



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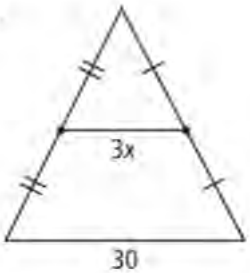

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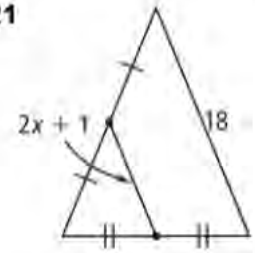

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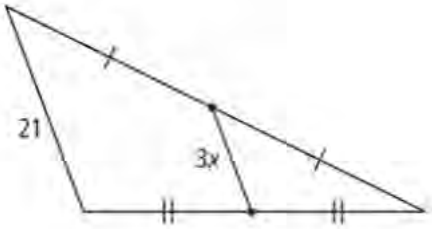
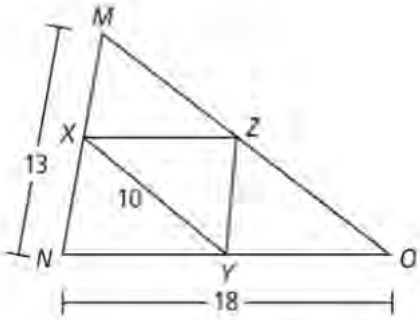


***X* is the midpoint of $\overline{MN}$. *Y* is the midpoint of $\overline{ON}$.**

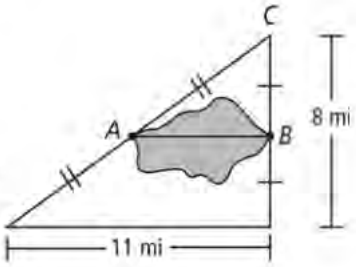


23. Find *XZ*.

24. If $XY = 10$, find *MO*.

25. If $m\square M$ is 64, find $m\square Y$.

Use the diagram at the right for Exercises 26 and 27.



26. What is the distance across the lake?

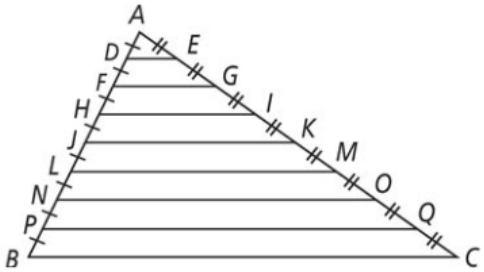
27. Is it a shorter distance from *A* to *B* or from *B* to *C*? Explain.

5-1 Enrichment

Midsegments of Triangles

Triangles and Maps

You can use the same reasoning behind the Triangle Midsegment Theorem to find the lengths of other line segments connecting the sides of a triangle.



Given the triangle at the right, write a paragraph proof for the following.

1. $FG = \frac{1}{4}BC$

2. $DE = \frac{1}{8}BC$

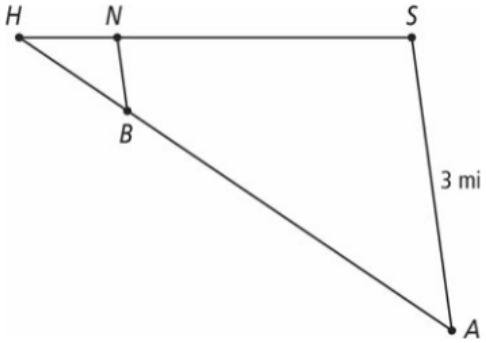
3. $NO = \frac{3}{4}BC$

4. $HI = \frac{3}{8}BC$

5. $LM = \frac{5}{8}BC$

6. $PQ = \frac{7}{8}BC$

Use the diagram at the right for Exercise 7.



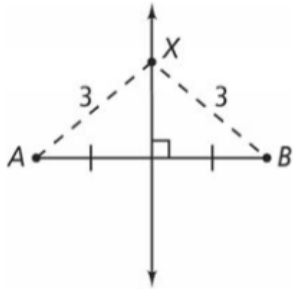
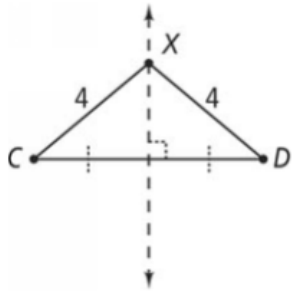
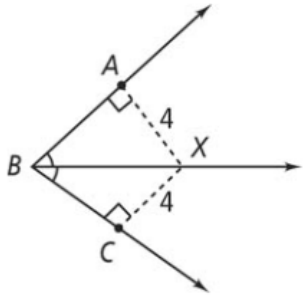
7. Nan is at point *N*, one-fourth of the way from her home at *H* to school at *S*. Bob is at *B*, which is three-quarters of the way from his apartment to Nan's home. Bob's apartment is at *A*, which is 3 mi from school. How far apart are Nan and Bob?

THE SCHOOL DISTRICT OF PHILADELPHIA

5-2 Additional Vocabulary Support

Perpendicular and Angle Bisectors

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Statement of Theorem	Picture or Example
Perpendicular Bisector Theorem	If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	
Converse of Perpendicular Bisector Theorem	**1.**	
Angle Bisector Theorem	**2.**	
Converse of Angle Bisector Theorem	If a point in the interior of an angle is equidistant from the sides of the angle, then the point is on the angle bisector.	**3.**

5-2 Reteaching

Perpendicular and Angle Bisectors

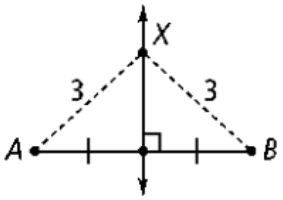
Perpendicular Bisectors

There are two useful theorems to remember about perpendicular bisectors.

Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

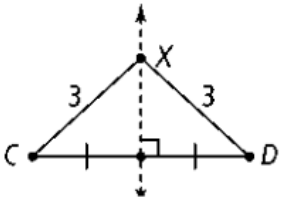
X is on the perpendicular bisector, so it is equidistant from the endpoints A and B.



Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

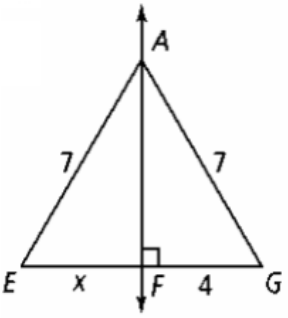
Because X is equidistant from the endpoints C and D, it is on the perpendicular bisector of the segment.



Problem

What is the value of x?

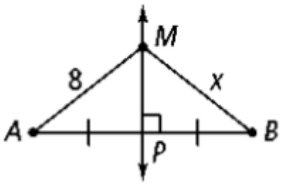
Since A is equidistant from the endpoints of the segment, it is on the perpendicular bisector of $\overline{EG}$. So, $EF = GF$ and $x = 4$.



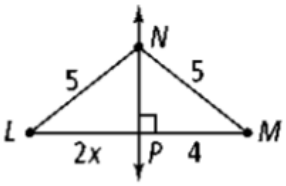
Exercises

Find the value of x.

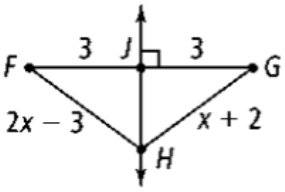
1.



2.



3.



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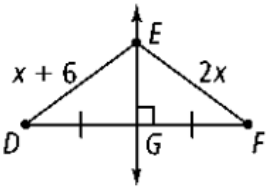

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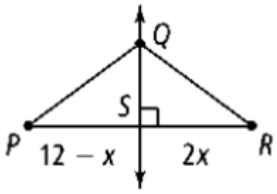
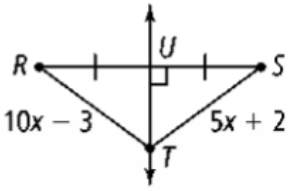


6.



5-2 Reteaching (continued)

Perpendicular and Angle Bisectors

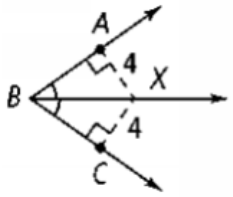
Angle Bisectors

There are two useful theorems to remember about angle bisectors.

Angle Bisector Theorem

If a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.

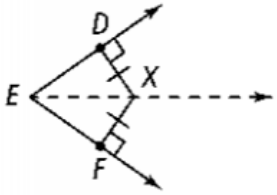
X is on the angle bisector and is therefore equidistant from the sides of the angle.



Converse of the Angle Bisector Theorem

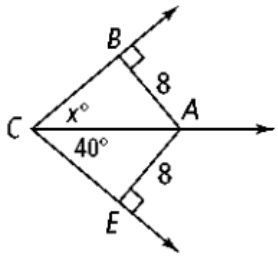
If a point in the interior of an angle is equidistant from the sides of an angle, then the point is on the angle bisector.

Because X is on the interior of the angle and is equidistant from the sides, X is on the angle bisector.



Problem

What is the value of x?



Because point A is in the interior of the angle and it is equidistant from the sides of the angle, it is on the bisector of the angle.

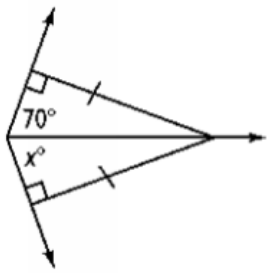
$$\angle BCA \cong \angle ECA$$

$$x = 40$$

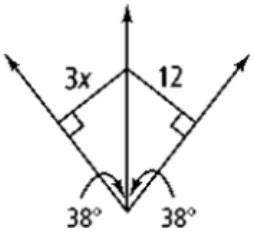
Exercises

Find the value of x.

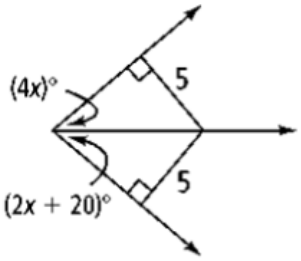
7.



8.



9.



THE SCHOOL DISTRICT OF PHILADELPHIA

5-2 Think About a Plan

Perpendicular and Angle Bisectors

a. **Constructions** Draw a large acute scalene triangle, □*PQR*. Construct the perpendicular bisectors of each side.

b. **Make a Conjecture** What appears to be true about the perpendicular bisectors?

c. Test your conjecture with another triangle.

1. For part (a), what is an acute scalene triangle?

_____

2. Sketch a large acute scalene triangle. Use a protractor to make sure each angle is less than 90°. Label the vertices *P*, *Q*, and *R*. Check to make sure the triangle is scalene by comparing the side lengths.

3. To construct the perpendicular bisector for $\overline{PQ}$, set the compass to greater than. ——— Draw two arcs, one from *P* and one from *Q*. The arcs _____ at two points. Draw a segment connecting the points. This segment is the _____ .

4. Construct the perpendicular bisectors of $\overline{QR}$ and $\overline{RP}$.

5. For part (b), examine the three perpendicular bisectors. Write a conjecture about the perpendicular bisectors in all triangles.

_____

_____

_____

6. For part (c), repeat Steps 1–4 for an obtuse, equilateral, or isosceles triangle. Does the conjecture appear to be true for this triangle?

_____

_____

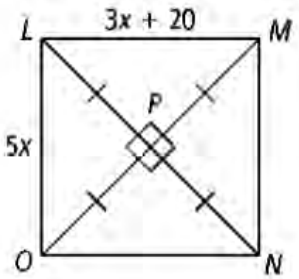
_____

5-2 Practice

Form G

Perpendicular and Angle Bisectors

Use the figure at the right for Exercises 1–4.



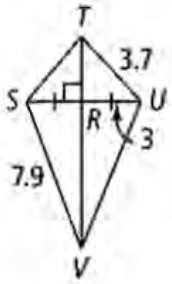
1. What is the relationship between $\overline{LN}$ and $\overline{MO}$?

2. What is the value of x?

3. Find LM.

4. Find LO.

Use the figure at the right for Exercises 5–8.



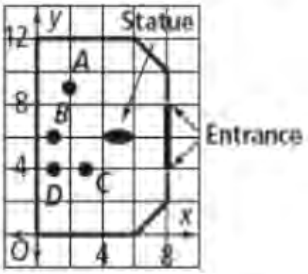
5. From the information given in the figure, how is $\overline{TV}$ related to $\overline{SU}$?

6. Find TS.

7. Find UV.

8. Find SU.

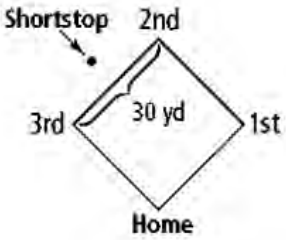
9. At the right is a layout for the lobby of a building placed on a coordinate grid.



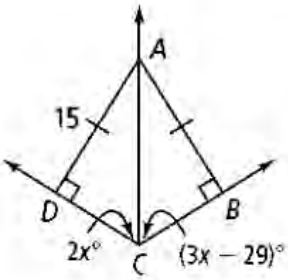
a. At which of the labeled points would a receptionist chair be equidistant from both entrances?

b. Is the statue equidistant from the entrances? How do you know?

10. In baseball, the baseline is a segment connecting the bases. A shortstop is told to play back 3 yd from the baseline and exactly the same distance from second base and third base. Describe how the shortstop could estimate the correct spot. There are 30 yd between bases. Assume that the shortstop has a stride of 36 in.



Use the figure at the right for Exercises 11–15.



11. According to the figure, how far is A from $\overline{CD}$? from $\overline{CB}$?

12. How is $\overrightarrow{CA}$ related to $\angle DCB$? Explain.

13. Find the value of x.

14. Find $m\angle ACD$ and $m\angle ACB$.

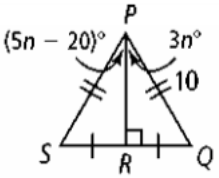
15. Find $m\angle DAC$ and $m\angle BAC$.

5-2 Practice (continued)

Form G

Perpendicular and Angle Bisectors

Use the figure at the right for Exercises 16–19.



16. According to the diagram, what are the lengths of $\overline{PQ}$ and $\overline{PS}$?

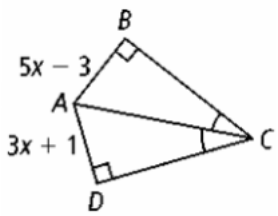
17. How is $\overline{PR}$ related to □SPQ?

18. Find the value of n.

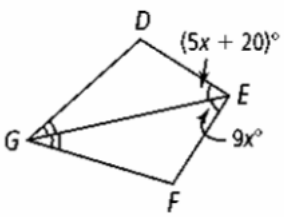
19. Find m□SPR and m□QPR.

Algebra Find the indicated variables and measures.

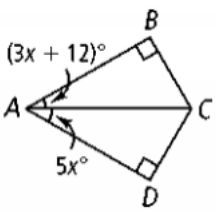
20. x, BA, DA



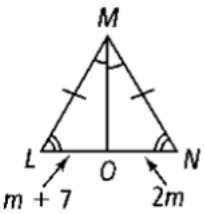
21. x, m□DEF



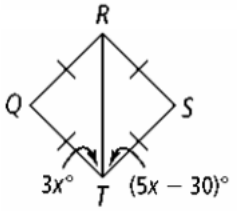
22. x, m□DAB



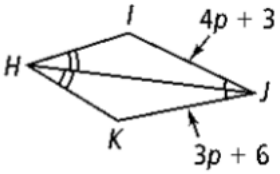
23. m, LO, NO



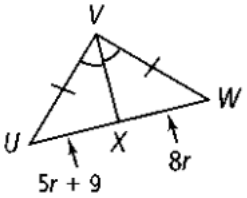
24. x, m□QTS



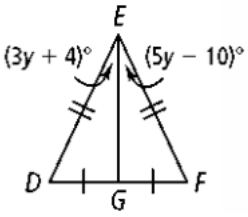
25. p, IJ, KJ



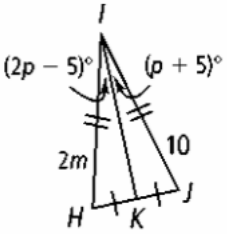
26. r, UW



27. y, m□DEF

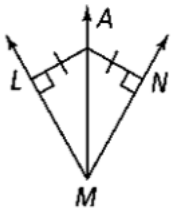


28. m, p

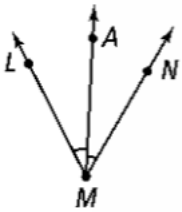


Writing Determine whether A must be on the bisector of □LMN. Explain.

29.



30.

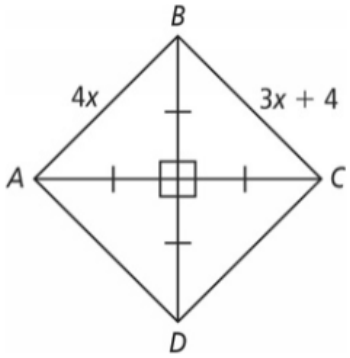


5-2 Practice

Form K

Perpendicular and Angle Bisectors

Use the figure at the right for Exercises 1–3.



1. What is the value of x?

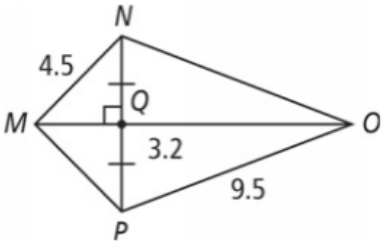
To start, determine the relationship between $\overline{AC}$ and $\overline{BD}$. Then write an equation to show the relationships of the sides. $\overline{BD}$ is the __?__ bisector of $\overline{AC}$. Therefore, point B is equidistant from points A and $\square$.

$4x =$ __?__

2. Find AB.

3. Find BC.

Use the figure at the right for Exercises 4–7.



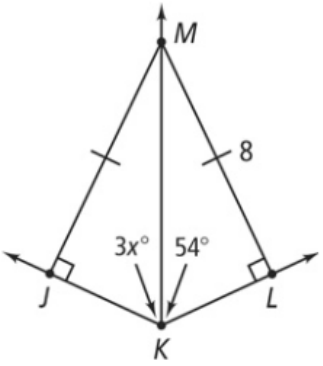
4. $\overline{MO}$ is the perpendicular bisector of $\square$.

5 Find MP.

6 Find NO.

7 Find NP.

Use the figure at the right for Exercises 8–13.



8. How far is M from $\overline{KL}$?

9. How far is M from $\overline{JK}$?

10. How is $\overline{KM}$ related to $\square JKL$?

11 Find the value of x.

12 Find $m\square MKL$.

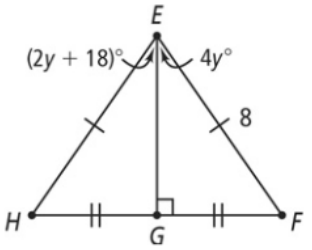
13 Find $m\square JMK$ and $m\square LMK$.

5-2 Practice (continued)

Form K

Perpendicular and Angle Bisectors

Use the figure at the right for Exercises 14–16.



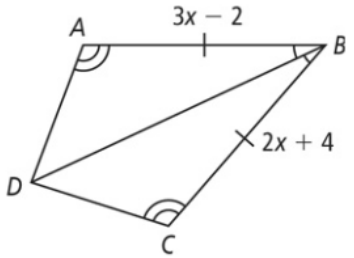
14. what are the lengths of $\overline{EF}$ and $\overline{EH}$?

15. Find the value of y.

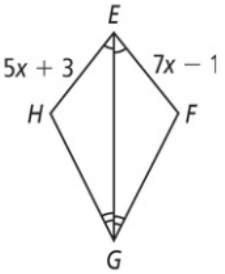
16. Find $m\square GEH$ and $m\square GEF$.

Algebra Find the values of the indicated variables and measures.

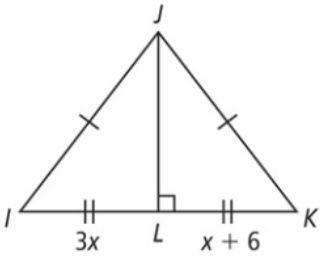
17. x, BA, BC



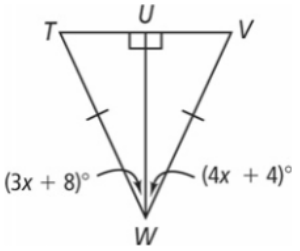
18. x, EH, EF



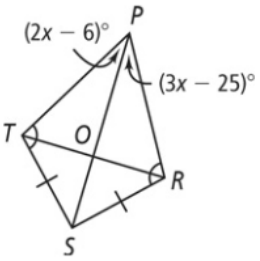
19. x, IK



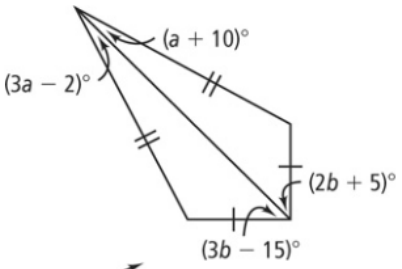
20. x, $m\square UWV$, $m\square UWT$



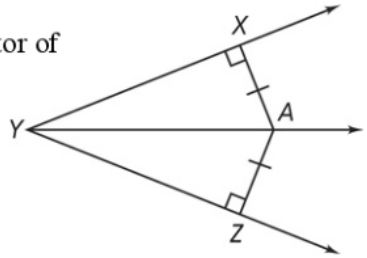
21. x, $m\square TPS$, $m\square RPS$



22. a, b



23. Writing Is A on the angle bisector of $\square XYZ$? Explain.



5-2 Enrichment

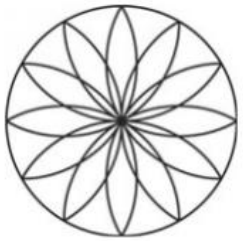
Perpendicular and Angle Bisectors

Angle Bisectors and Daisy Designs

Materials

- Compass
- Straightedge

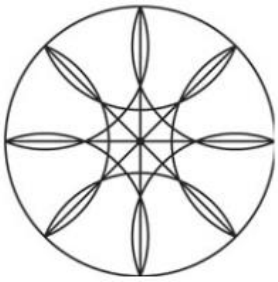
Follow the directions to replicate the daisy design.



1. Use a compass to construct a circle.
2. Pick a point on the circle, keeping the radius the same, and mark a full arc that intersects the circle.
3. From the intersection point of the arc and circle, make another full arc and continue around the circle.
4. Create a point at each intersection.
5. Using two consecutive points on the circle and the center of the circle, draw an angle with the vertex at the center of the circle.
6. Bisect that angle.
7. Mark the point of intersection with the circle.
8. Using this point and the original radius of the circle, make an arc and continue around the circle. Create a point at each intersection.
9. Starting from any point, draw full arcs to connect two points on the circle.
10. Continue around the circle. When the figure is complete, erase any unnecessary marks.

Use your knowledge of straightedge and compass constructions to create the daisy design below.

11.



5-3 Additional Vocabulary Support

Bisectors in Triangles

For Exercises 1–5, match the term in Column A with its description in Column B. The first one is done for you.

Column A	Column B
concurrent	the point of intersection of three or more lines
1. point of concurrency	the intersection point of the three angle bisectors of a triangle
2. circumcenter of a triangle	when a circle is tangent to the three sides of a triangle
3. circumscribed about	when three or more lines intersect at a single point
4. incenter of a triangle	when a circle passes through the three vertices of a triangle
5. inscribed in	the intersection point of the three perpendicular bisectors of a triangle

For Exercises 6–8, match the phrase in Column A with the diagram in Column B that describes point *P*.

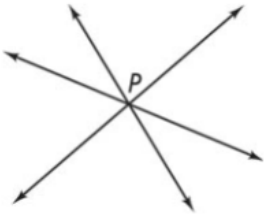
Column A

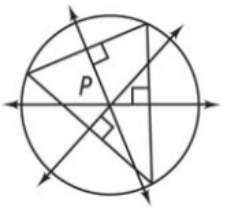
6. circumcenter of a triangle

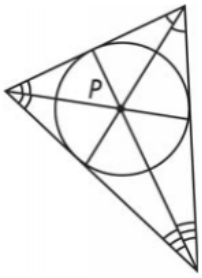
7. point of concurrency

8. incenter of a triangle

Column B





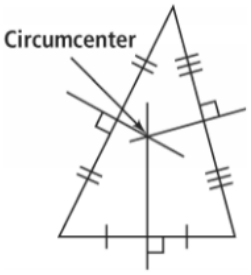


5-3 Reteaching

Bisectors in Triangles

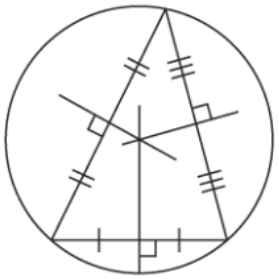
The Circumcenter of a Triangle

If you construct the perpendicular bisectors of all three sides of a triangle, the constructed segments will all intersect at one point. This point of concurrency is known as the circumcenter of the triangle.



It is important to note that the circumcenter of a triangle can lie inside, on, or outside the triangle.

The circumcenter is equidistant from the three vertices. Because of this, you can construct a circle centered on the circumcenter that passes through the triangle's vertices. This is called a *circumscribed* circle.

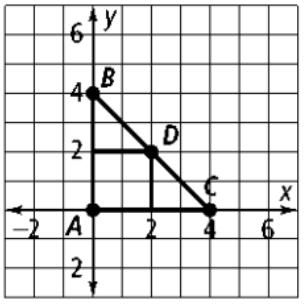


Problem

Find the circumcenter of right ΔABC.

First construct perpendicular bisectors of the two legs, $\overline{AB}$ and $\overline{AB}$. These intersect at (2, 2), the circumcenter.

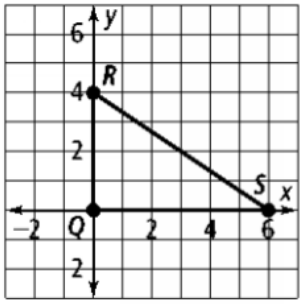
Notice that for a right triangle, the circumcenter is on the hypotenuse.



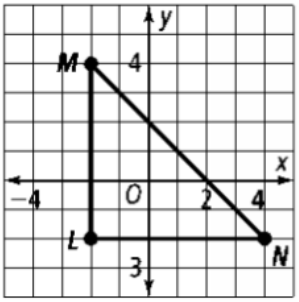
Exercises

Coordinate Geometry **Find the circumcenter of each right triangle.**

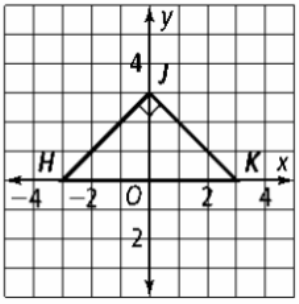
1



2



3.



Coordinate Geometry **Find the circumcenter of ΔABC.**

4. $A(0, 0)$
$B(0, 8)$
$C(10, 8)$

5. $A(27, 3)$
$B(9, 3)$
$C(27, 27)$

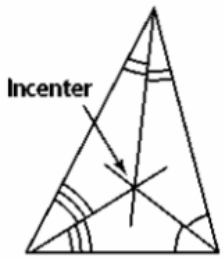
6. $A(25, 2)$
$B(3, 2)$
$C(3, 6)$

5-3 Reteaching (continued)

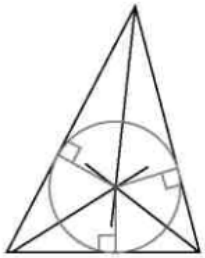
Bisectors in Triangles

The Incenter of a Triangle

If you construct angle bisectors at the three vertices of a triangle, the segments will intersect at one point. This point of concurrency where the angle bisectors intersect is known as the *incenter of the triangle*.



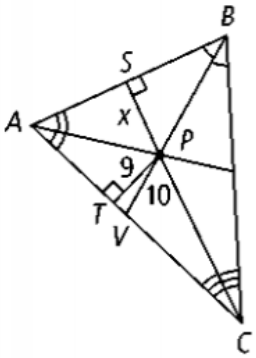
It is important to note that the incenter of a triangle will always lie inside the triangle.



The incenter is equidistant from the sides of the triangle. You can draw a circle centered on the incenter that just touches the three sides of the triangle. This is called an *inscribed* circle.

Problem

Find the value of x.



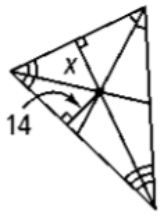
The angle bisectors intersect at P. The incenter P is equidistant from the sides, so $SP = PT$. Therefore, $x = 9$.

Note that $\overline{PV}$, the continuation of the angle bisector, is not the correct segment to use for the shortest distance from P to $\overline{AC}$.

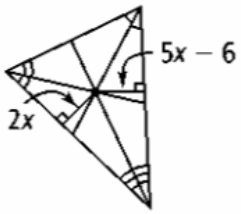
Exercises

Find the value of x.

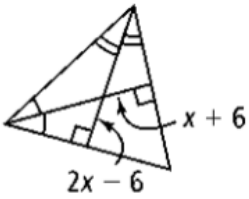
7.



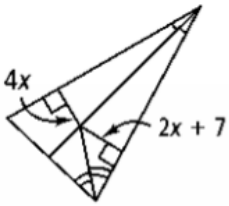
8.



9.



10.



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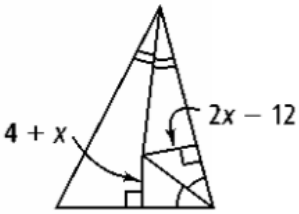
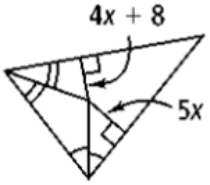


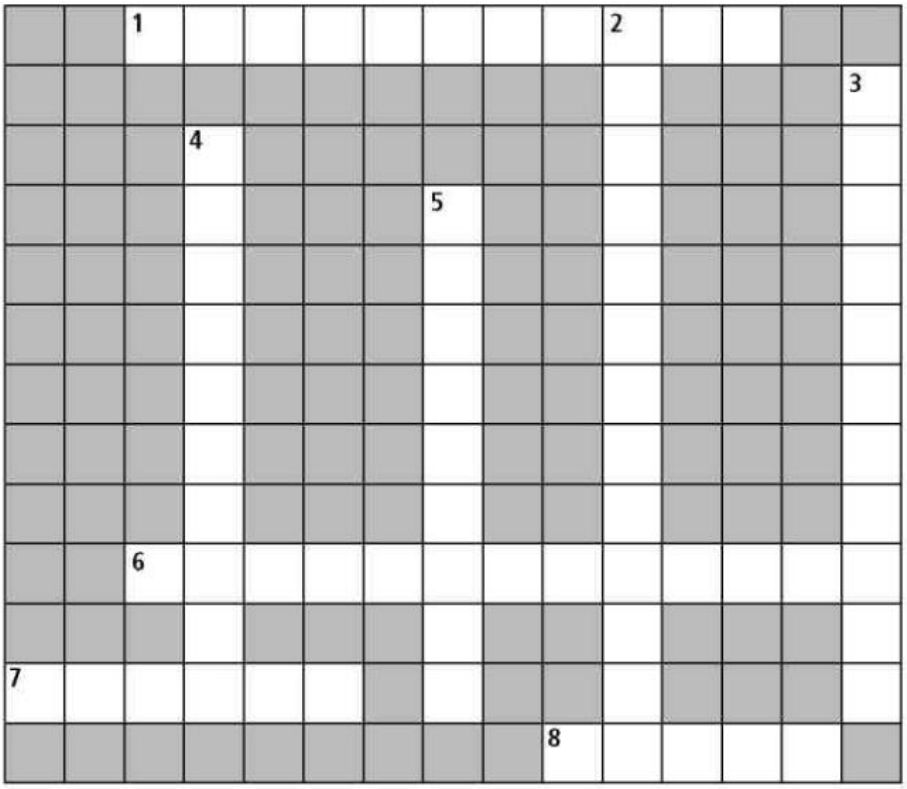
12.



5-3 Puzzle: Crossword

Bisectors in Triangles

Each clue below involves a vocabulary word you have used or will use soon. Find the missing word or words to complete each sentence. Then place your answers in the crossword puzzle.



Across

1. A point is __?__ from two objects if it is the same distance from the objects.
6. The distance from a point to a line is the length of the __?__ segment from the point to the line.
7. The point where two or more sides of a figure meet is called a(n) __?__ .
8. A convincing argument that logically shows why a conjecture is true is called a(n) __?__ .

Down

2. Given the point D in the interior of $\square ABC$, if $m\square ABC = 40\square$ and $m\square DBC = 20\square$, then $\overline{BD}$ is called a(n) __?__ (two words).
3. A(n) __?__ triangle has all congruent sides.
4. A segment that connects the midpoints of two sides of a triangle is called a(n) __?__ .
5. If a point is on the perpendicular bisector of a segment, then it is equidistant from the __?__ of the segment.

5-3 Think About a Plan

Bisectors in Triangles

Writing Ivars found an old piece of paper inside an antique book. It read:

From the spot I buried Olaf's treasure, equal sets of paces did I measure; each of three directions in a line, there to plant a seedling Norway pine. I could not return for failing health; now the hounds of Haiti guard my wealth. —Karl

After searching Caribbean islands for five years, Ivars found an island with three tall Norway pines. How might Ivars find where Karl buried Olaf's treasure?

Know

1. Make a sketch as you answer the questions.
2. *"From the spot I buried Olaf's treasure ..."* Mark a point X on your paper.
3. *"... equal sets of paces I did measure; each of the three directions in a line ..."* This tells you to draw segments that have an endpoint at X.

   a. Explain how you know these are segments. _____

   b. How many segments should you draw?

   c. What do you know about the length of the segments? _____

   d. What do you know about the endpoints of the segments?

   _____

4. You do not know in which direction to draw each segment, but you can choose three directions for your sketch. Mark the locations of the trees. Draw a triangle with the trees at its vertices. What is the name of the point where X is located? _____

Need

5. Look at your sketch. What do you need to find? _____

Plan

6. Describe how to find the treasure. The first step is done for you.

   Step 1 Find the midpoints of each side of the triangle.

   Step 2 _____

   Step 3 _____

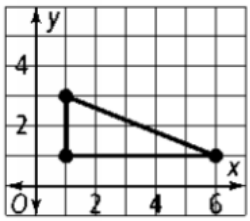
5-3 Practice

Form G

Bisectors in Triangles

Coordinate Geometry **Find the circumcenter of each triangle.**

1.



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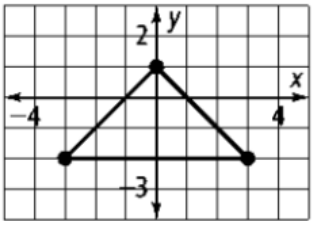

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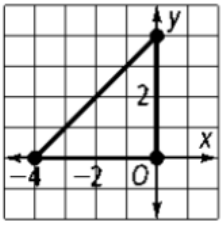


Coordinate Geometry **Find the circumcenter of □*ABC*.**

4. $A(1, 3)$
$B(4, 3)$
$C(4, 2)$

5. $A(2, 3)$
$B(4, 3)$
$C(4, 7)$

6. $A(5, 2)$
$B(1, 2)$
$C(1, 6)$

7. $A(5, 6)$
$B(0, 6)$
$C(0, 3)$

8. $A(1, 3)$
$B(5, 3)$
$C(5, 2)$

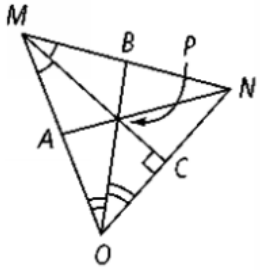
9. $A(2, 2)$
$B(4, 2)$
$C(4, 7)$

10. $A(5, 3)$
$B(1, 3)$
$C(1, 6)$

11. $A(5, 2)$
$B(1, 2)$
$C(1, 3)$

Name the point of concurrency of the angle bisectors.

12.



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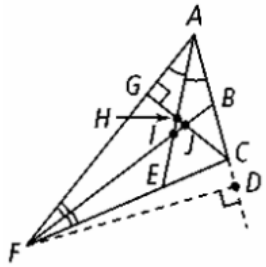

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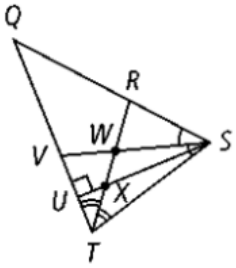

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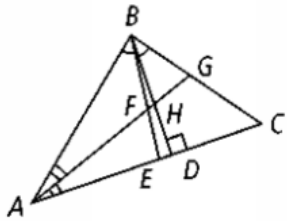

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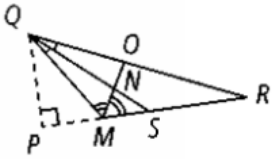

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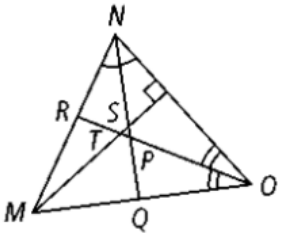


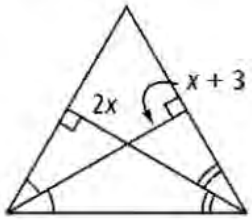
5-3 Practice (continued)

Form G

Bisectors in Triangles

Find the value of *x*.

18.



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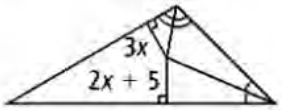

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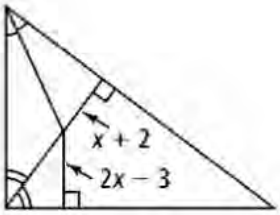
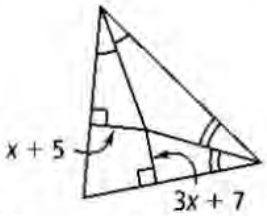


21.



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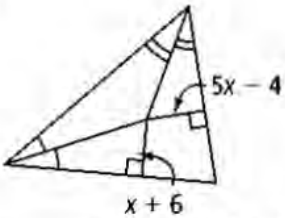

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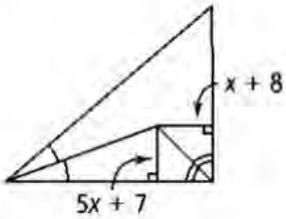
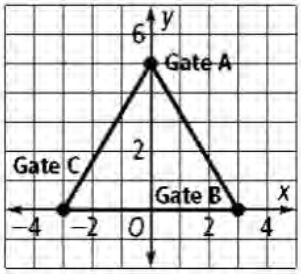
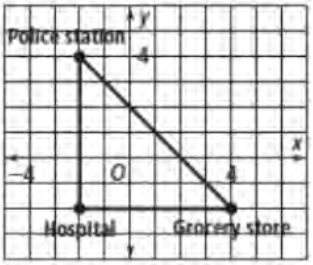


24. Where should the farmer place the hay bale so that it is equidistant from the three gates?

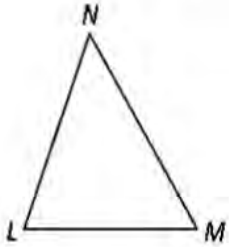


25. Where should the fire station be placed so that it is equidistant from the grocery store, the hospital, and the police station?

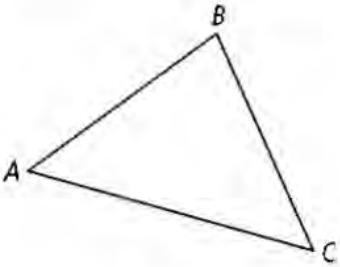


26. Construction Construct three perpendicular bisectors for □*MN*. Then use the point of concurrency to construct the circumscribed circle.

00



27. Construction Construct two angle bisectors for □*ABC*. Then use the point of concurrency to construct the inscribed circle.



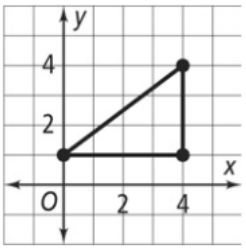
5-3 Practice

Form K

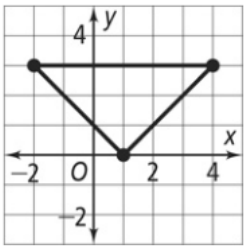
Bisectors in Triangles

Coordinate Geometry **Find the coordinates of the circumcenter of each triangle.**

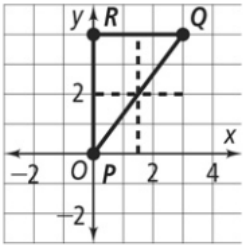
1.



2.



Coordinate Geometry **Find the circumcenter of ΔPQR.**



3. P (0, 0)
Q (3, 4)
R (0, 4)

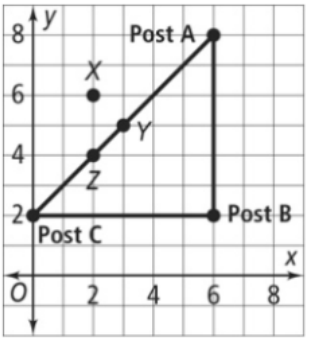
To start, graph the vertices and connect them on a coordinate plane. Then draw two perpendicular bisectors.

4. P (1, 5)
Q (4, 5)
R (1, 2)

5. P (3, 5)
Q (3, 2)
R (1, 5)

6. P (6, 6)
Q (3, 6)
R (6, 2)

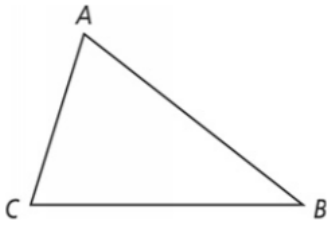
7. P (4, 6)
Q (1, 6)
R (1, 2)



8. a. Which point is equidistant from the three posts?

b. Where are the coordinates of this point?

9. Construction Construct three perpendicular bisectors for ΔABC. Then use the point of concurrency to construct the circumscribed circle.



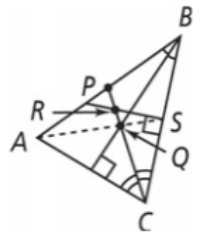
5-3 Practice (continued)

Form K

Bisectors in Triangles

Name the point of concurrency of the angle bisectors.

10.



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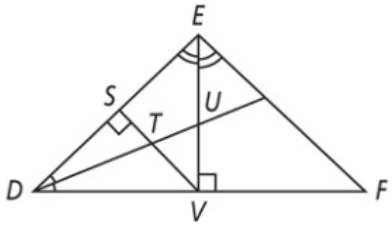
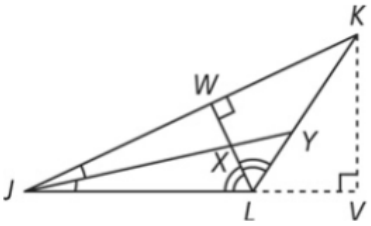
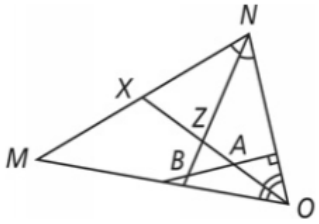


12.



13.



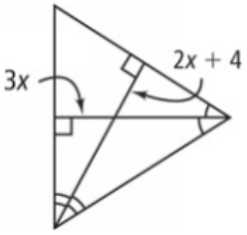
Find the value of *x*.

14. To start, identify the relationship between the line segments that are labeled.

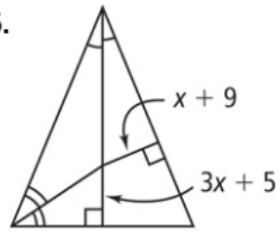
Because the segments meet at the point where the ? meet, the segments are ? .

Then write an equation to find *x*:

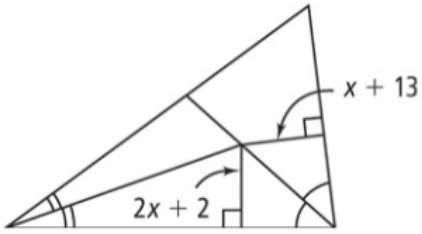
☐ = ☐ + ☐



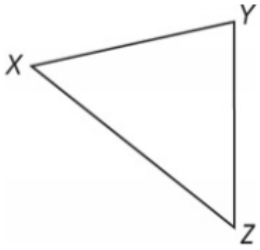
15.



16.



17. Construction Construct two angle bisectors for ΔXYZ. Then use the point of concurrency to construct the inscribed circle.



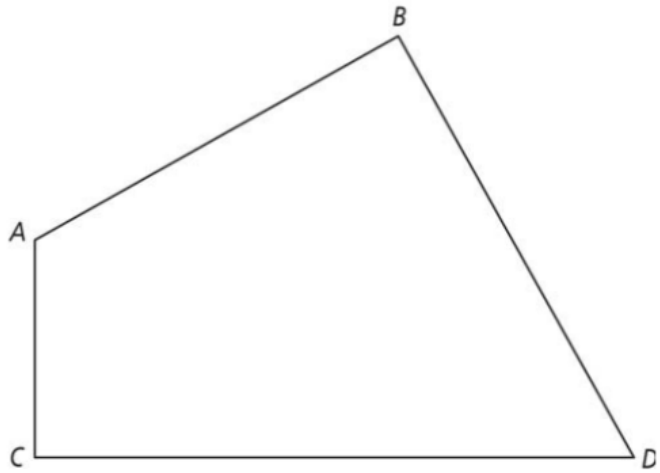
5-3 Enrichment

Bisectors in Triangles

Circumcenter of a Quadrilateral

While all triangles have a circumcenter, not all polygons have circumcenters. Quadrilaterals that have circumcenters are called *cyclic quadrilaterals,* because there is a circle that goes through all four vertices. Here you will construct the circumcenter of a given cyclic quadrilateral.

Use a compass and straightedge to perform the following construction for the circumcenter of quadrilateral *ABDC*.



1. Choose a point inside quadrilateral *ABDC*. Label it point *P*.
2. Construct perpendicular bisectors to find the circumcenter of ΔCAP. Label the circumcenter *E*.
3. Construct the perpendicular bisectors of ΔABP. Label the circumcenter *F*.
4. Construct the perpendicular bisectors of ΔBDP. Label the circumcenter *G*.
5. Construct the perpendicular bisectors of ΔDCP. Label the circumcenter *H*.
6. Draw diagonals $\overline{AD}$ and $\overline{BC}$. Label their intersection point *I*.
7. Draw $\overleftrightarrow{EG}$ and $\overleftrightarrow{FH}$. Label their intersection point *K*.
8. Draw $\overleftrightarrow{KI}$.
9. Mark a point on $\overleftrightarrow{KI}$ that is the same distance from *K* as is *I*, but on the other side of *K*. Label this *O*. This is the circumcenter.
10. To draw a circle that circumscribes quadrilateral *ABDC*, put your compass point at *O* and your pencil on one of the vertices and draw a circle through the vertices of the quadrilateral.

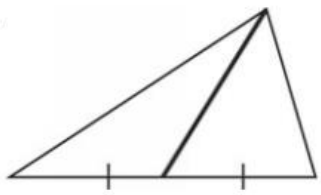
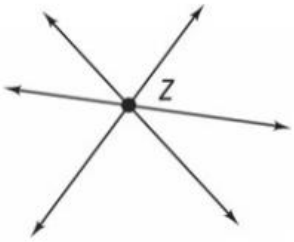
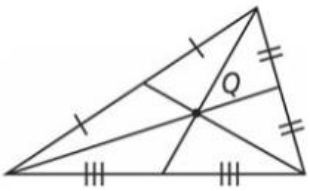
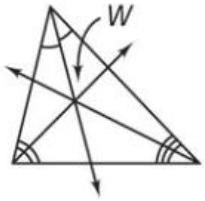
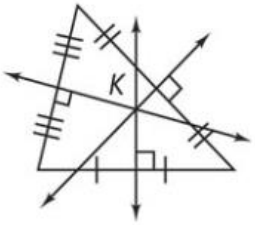
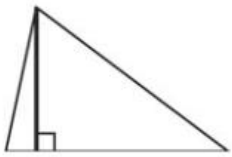
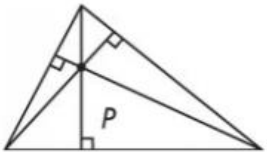
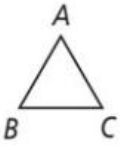
5-4 Additional Vocabulary Support

Medians and Altitudes

Concept List

altitude	centroid	circumcenter
concurrent lines	incenter	median
orthocenter	point of concurrency	vertex

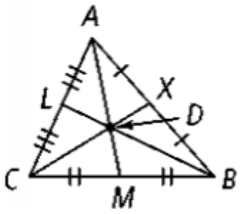
Choose the concept from the list above that best represents the item in each box.

1. 	**2.** Point Z 	**3.** Point Q 
4. Point W 	**5.** Three or more lines that meet at a single point	**6.** Point K 
7. 	**8.** Point P 	**9.** Point A, B, or C 

5-4 Reteaching

Medians and Altitudes

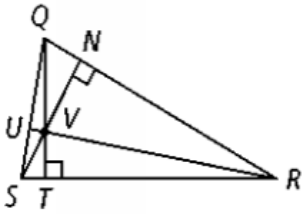
A *median* of a triangle is a segment that runs from one vertex of the triangle to the midpoint of the opposite side. The point of concurrency of the medians is called the *centroid*.



The medians of ΔABC are $\overline{AM}$, $\overline{CX}$, and $\overline{BL}$.

The centroid is point D.

An *altitude* of a triangle is a segment that runs from one vertex perpendicular to the line that contains the opposite side. The *orthocenter* is the point of concurrency for the altitudes. An altitude may be inside or outside the triangle, or a side of the triangle.

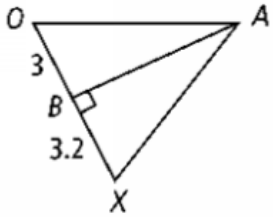


The altitudes of ΔQRS are $\overline{QT}$, $\overline{RU}$, and $\overline{SN}$.

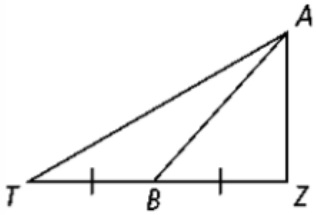
The orthocenter is point V.

Determine whether $\overline{AB}$ is a *median*, an *altitude*, or *neither*.

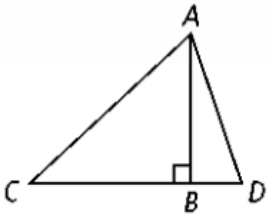
1.



2.



3.



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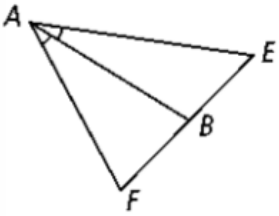
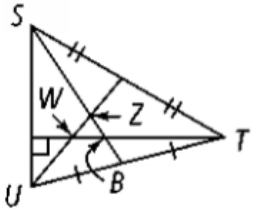
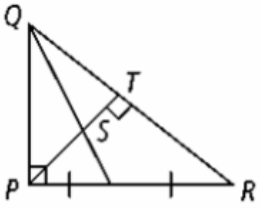


5. Name the centroid.



6. Name the orthocenter.



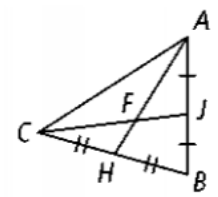
5-4 Reteaching (continued)

Medians and Altitudes

The medians of a triangle intersect at a point two-thirds of the distance from a vertex to the opposite side. This is the Concurrency of Medians Theorem.

$\overline{CJ}$ and $\overline{AH}$ are medians of ΔABC and point F is the centroid.

$$CF = \frac{2}{3}CJ$$



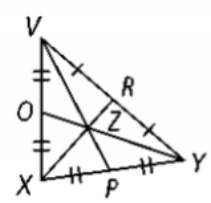
Problem

Point F is the centroid of ΔABC. If $CF = 30$, what is CJ?

$CF = \frac{2}{3}CJ$	Concurrency of Medians Theorem
$30 = \frac{2}{3} \cdot CJ$	Fill in known information.
$\frac{3}{2} \cdot 30 = CJ$	Multiply each side by $\frac{3}{2}$.
$45 = CJ$	Solve for CJ.

Exercises

In ΔVYX, the centroid is Z. Use the diagram to solve the problems.



7. If $XR = 24$, find XZ and ZR.

8. If $XZ = 44$, find XR and ZR.

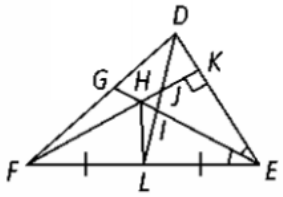
9. If $VZ = 14$, find VP and ZP.

10. If $VP = 51$, find VZ and ZP.

11. If $ZO = 10$, find YZ and YO.

12. If $YO = 18$, find YZ and ZO.

In Exercises 13–16, name each segment.



13. a median in ΔDEF

14. an altitude in ΔDEF

15. a median in ΔEHF

16. an altitude in ΔHEK

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5-4 Think About a Plan

Medians and Altitudes

Coordinate Geometry □ABC has vertices $A(0, 0)$, $B(2, 6)$, and $C(8, 0)$. Define the points L, M, and N such that $AL = LB$, $BM = MC$, and $CN = NA$. Complete the following steps to verify the Concurrency of Medians Theorem for □ABC.

a. Find the coordinates of midpoints L, M, and N.

b. Find equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

c. Find the coordinates of P, the intersection of $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$. This is the centroid.

d. Show that point P is on $\overleftrightarrow{CL}$.

e. Use the Distance Formula to show that point P is two-thirds of the distance from each vertex to the midpoint of the opposite side.

1. Write the midpoint formula.

2. Use the formula to find the coordinates of L, M, and N.

3. Find the slopes of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

4. Write the general point-slope form of a linear equation.

5. Write the point-slope form equations of $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$, and $\overleftrightarrow{CL}$.

6. Solve the system of equations for $\overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$ to find the point of intersection.

7. Show that the coordinates of point P satisfy the equation of $\overleftrightarrow{CL}$.

8. Use the distance formula to find AM, BN, and CL. Use a calculator and round to the nearest hundredth.

9. Use the distance formula to find AP, BP, and CP.

10. Check to see that $AP = \frac{2}{3}AM$, $BP = \frac{2}{3}BN$, and $CP = \frac{2}{3}CL$.

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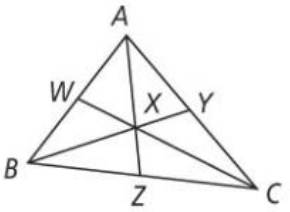
5-4 Practice

Form G

Medians and Altitudes

In □*ABC*, *X* is the centroid.



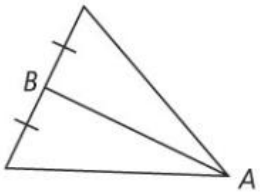
1. If $CW = 15$, find CX and XW.

2. If $BX = 8$, find BY and XY.

3. If $XZ = 3$, find AX and AZ.

Is $\overline{AB}$ a *median*, an *altitude*, or *neither?* Explain.

4.



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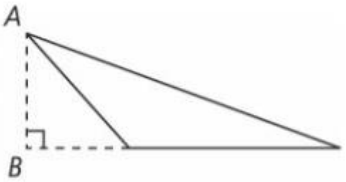

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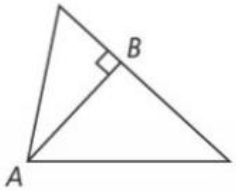
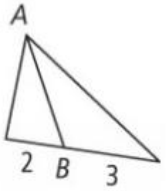


7.

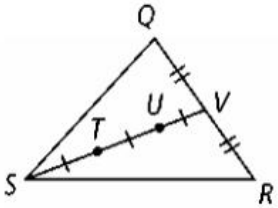


Coordinate Geometry Find the orthocenter of □*ABC*.

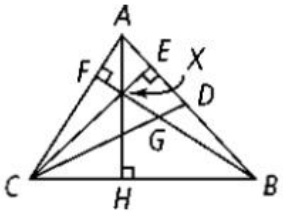
8. $A(2, 0)$, $B(2, 4)$, $C(6, 0)$

9. $A(1, 1)$, $B(3, 4)$, $C(6, 1)$

10. Name the centroid.



11. Name the orthocenter.



Draw a triangle that fits the given description. Then construct the centroid and the orthocenter.

12. equilateral □*CDE*

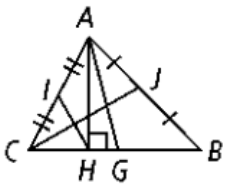
13. acute isosceles □*XYZ*

5-4 Practice (continued)

Form G

Medians and Altitudes

In Exercises 14–18, name each segment.



14. a median in □*ABC*

15. an altitude for □*ABC*

16. a median in □*AHC*

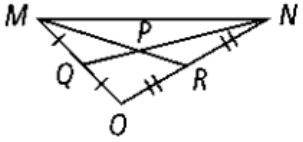
17. an altitude for □*AHB*

18. an altitude for □*AHG*

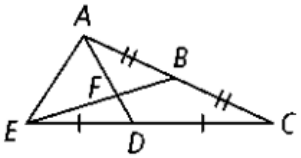
19. $A(0, 0)$, $B(0, 2)$, $C(3, 0)$. Find the orthocenter of □*ABC*.

20. Cut a large isosceles triangle out of paper. Paper-fold to construct the medians and the altitudes. How are the altitude to the base and the median to the base related?

21. In which kind of triangle is the centroid at the same point as the orthocenter?



22. P is the centroid of □*MNO*. $MP = 14x + 8y$. Write expressions to represent PR and MR.



23. F is the centroid of □*ACE*. $AD = 15x^2 + 3y$. Write expressions to represent AF and FD.

24. Use coordinate geometry to prove the following statement.

Given: □*ABC*; $A(c, d)$, $B(c, e)$, $C(f, e)$

Prove: The circumcenter of □*ABC* is a point on the triangle.

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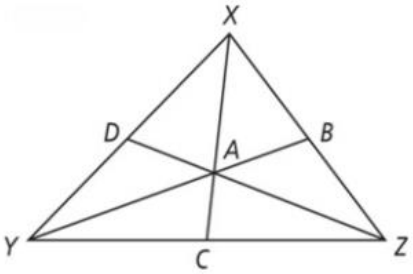
5-4 Practice

Form K

Medians and Altitudes

In $\triangle XYZ$, A is the centroid.



1. If $DZ = 12$, find ZA and AD.

To start, write an equation relating the distance between the vertex and centroid to the length of the median.

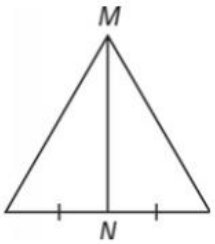
$ZA = \square\, DZ$

2. If $AB = 6$, find BY and AY.

3. If $AC = 3$, find CX and AX.

Is $\overline{MN}$ a *median*, an *altitude*, or *neither?* Explain.

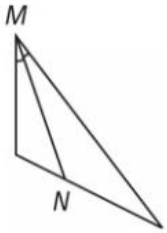
4.



To start, identify the relationship between $\overline{MN}$ and the side it intersects.

$\overline{MN}$? the side of the triangle it intersects.

5.



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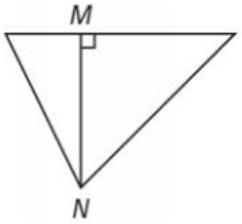
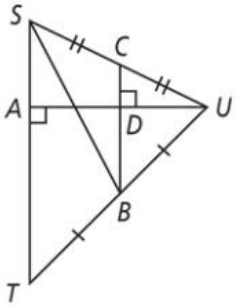


In Exercises 7–10, name each segment.



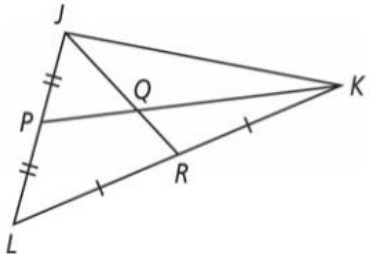
7. a median in ΔSTU

8. an altitude in ΔSTU

9. a median in ΔSBU

10. an altitude in ΔCBU

11. Q is the centroid of ΔJKL. $PK = 9x + 21y$. Write expressions to represent PQ and QK.



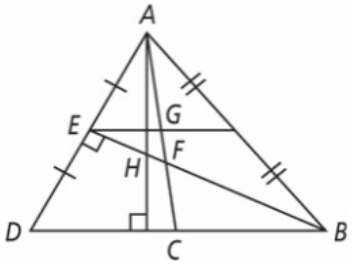
5-4 Practice (continued)

Form K

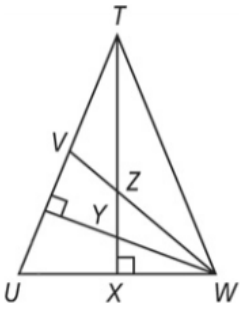
Medians and Altitudes

Find the orthocenter of each triangle.

12.



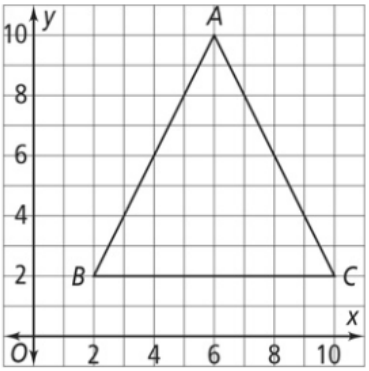
13.



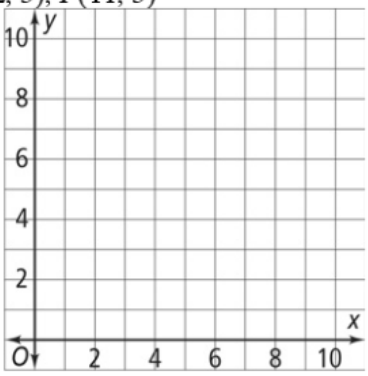
Coordinate Geometry **Find the coordinates of the orthocenter of ΔABC.**

14. $A(6, 10)$, $B(2, 2)$, $C(10, 2)$

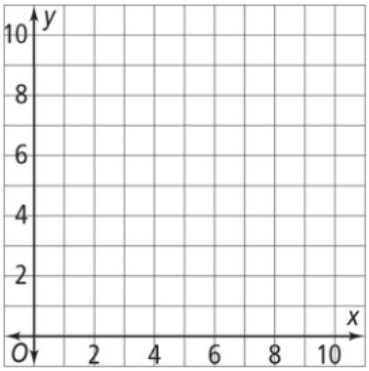
To start, graph the vertices of the triangle in a coordinate plane.



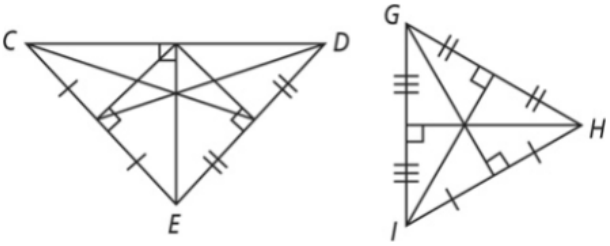
15. $P(1, 7)$, $Q(1, 2)$, $R(11, 2)$
$E(2, 5)$, $F(11, 5)$



16. $D(5, 11)$,



17. Which triangle has a centroid at the same point as the orthocenter?



5-4 Enrichment

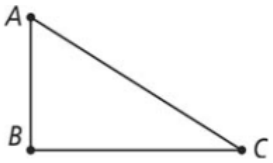
Medians and Altitudes

Constructions — Centroid and Orthocenter

Materials

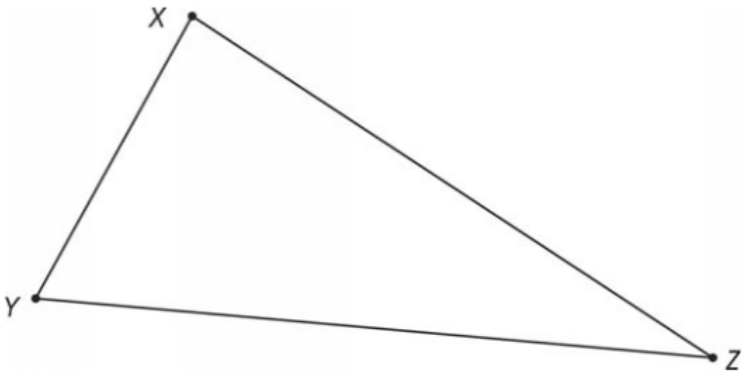
- Compass
- Straightedge
- Card stock

Follow the directions below to locate a point of concurrency of ΔABC.



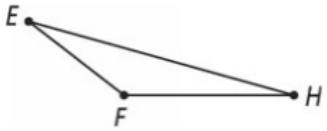
1. Using a construction or folding, find the midpoint of each side of the triangle.
2. Draw all three medians of the triangle.
3. What word names the point of intersection of three medians? Label this point D.
4. Is this point in the interior or the exterior of ΔABC?
5. Is it possible for this point to lie outside a triangle? Explain.

Trace ΔXYZ on a piece of card stock or cardboard, and cut it out.



6. Find the balancing point of the triangle using constructions.
7. Test your answer by placing the point on the tip of a pencil and observing whether the triangle is balanced.
8. Did you find the circumcenter, incenter, centroid, or orthocenter of the triangle?

Follow the directions below to locate a point of concurrency of ΔEFH.



9. Construct all three altitudes of ΔEFH. (*Hint:* Remember that some altitudes may lie outside the triangle.)
10. What word names the point of intersection of the altitudes? Label this point I.
11. Construct two altitudes of ΔABC and ΔXYZ.
12. Where do the orthocenters for obtuse, right, and acute triangles lie?

THE SCHOOL DISTRICT OF PHILADELPHIA

5-5 Additional Vocabulary Support

Indirect Proof

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

Related Words	Explanations	Examples
Assume uh SOOM	to say or believe something to be true	The weather has been cold every day this week. I *assume* it will be cold today, too.
Assumption uh SUMP shun	something taken to be true, or believed to be true	My *assumption* is that the weather in Canada is cold.
Reason (noun) REE zun	a basis for a belief or an action	My *reason* for believing that the sun will rise tomorrow is that it has always risen before.
Reason (verb) REE zun	to think or argue logically; to form conclusions	Dirk *reasons* with his sister to get her to eat her vegetables.
Reasoning REE zun ing	way of thinking, analysis	inductive *reasoning*, deductive *reasoning*
Reasonable REE zuh nuh bul	to be logical, to make sense	After solving a problem, Maria checks to see if her answer is *reasonable*.

Circle the correct answer. The first one is done for you.

I [reason/assume] that people use umbrellas when it rains.

1. A student multiplies 21 × 43 to get 903. She checks to see if her answer is [reasonable/assumption] by comparing it to the product of 20 and 40.
2. A dad could not find his children's sandals. He made the [reason/assumption] that his children wore the sandals to the beach.

Use the vocabulary above to fill in the blanks.

3. Before he jumped into the pool, a swimmer made the _____ that the water would be warm.
4. A girl's _____ for believing that all cats purr is that every cat she knows purrs.
5. Everybody likes the chef's cooking. A diner concludes using inductive _____ that she will enjoy the dinner the chef is cooking tonight.

5-5 Reteaching

Indirect Proof

In an *indirect proof,* you prove a statement or conclusion to be true by proving the opposite of the statement to be false.

There are three steps to writing an indirect proof.

Step 1: State as a temporary assumption the opposite (negation) of what you want to prove.

Step 2: Show that this temporary assumption leads to a contradiction.

Step 3: Conclude that the temporary assumption is false and that what you want to prove must be true.

Problem

Given: There are 13 dogs in a show; some are long-haired and the rest are short-haired. There are more long-haired than short-haired dogs.

Prove: There are at least seven long-haired dogs in the show.

Step 1: Assume that fewer than seven long-haired dogs are in the show.

Step 2: Let ℓ be the number of long-haired dogs and s be the number of short-haired dogs. Because $\ell + s = 13$, $s = 13 - \ell$. If ℓ is less than 7, s is greater than or equal to 7. Therefore, s is greater than ℓ. This *contradicts* the statement that there are more long-haired than short-haired dogs.

Step 3: Therefore, there are at least seven long-haired dogs.

Exercises

Write the temporary assumption you would make as a first step in writing an indirect proof.

1. **Given:** an integer q; **Prove:** q is a factor of 34.
2. **Given:** ΔXYZ; **Prove:** $XY + XZ > YZ$.
3. **Given:** rectangle $GHIJ$; **Prove:** $m\square G = 90$
4. **Given:** $\overline{XY}$ and $\overline{XM}$; **Prove:** $XY = XM$

Write a statement that contradicts the given statement.

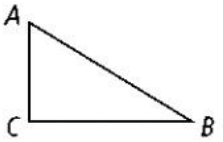
5. Whitney lives in an apartment.
6. Marc does not have three sisters.
7. $\square 1$ is a right angle.
8. Lines m and h intersect.

5-5 Reteaching (continued)

Problem

Given: $\angle A$ and $\angle B$ are not complementary.

Prove: $\angle C$ is not a right angle.



Step 1: Assume that $\angle C$ is a right angle.

Step 2: If $\angle C$ is a right angle, then by the Triangle Angle-Sum Theorem, $m\angle A + m\angle B + 90 = 180$. So $m\angle A + m\angle B = 90$. Therefore, $\angle A$ and $\angle B$ are complementary. But $\angle A$ and $\angle B$ are not complementary.

Step 3: Therefore, $\angle C$ is not a right angle.

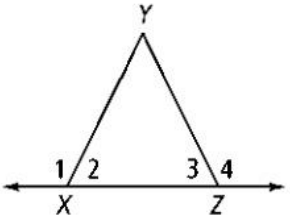
Exercises

Complete the proofs.

9. Arrange the statements given at the right to complete the steps of the indirect proof.

Given: $\overline{XY} \ncong \overline{YZ}$

Prove: $\angle 1 \ncong \angle 4$



Step 1: ?

Step 2: ?

Step 3: ?

Step 4: ?

Step 5: ?

Step 6: ?

A. But $\overline{XY} \ncong \overline{YZ}$.

B. Assume $\angle 1 \cong \angle 4$.

C. Therefore, $\angle 1 \ncong \angle 4$.

D. $\angle 1$ and $\angle 2$ are supplementary, and $\angle 3$ and $\angle 4$ are supplementary.

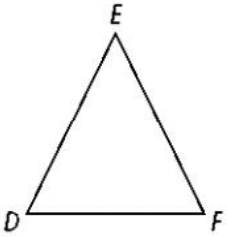
E. According to the Converse of the Isosceles Triangle Theorem, $XY = YZ$ or $\overline{XY} \cong \overline{YZ}$.

F. If $\angle 1 \cong \angle 4$, then by the Congruent Supplements Theorem, $\angle 2 \cong \angle 3$.

10. Complete the steps below to write a convincing argument using indirect reasoning.

Given: ΔDEF with $\angle D \ncong \angle F$

Prove: $\overline{EF} \ncong \overline{DE}$



Step 1: ?

Step 2: ?

Step 3: ?

Step 4: ?

5-5 Think About a Plan

Indirect Proof

Write an indirect proof.

Given: ΔXYZ is isosceles.

Prove: Neither base angle is a right angle.

1. What is the first step in writing an indirect proof?

_____

2. Write the first step for this indirect proof.

_____

3. What is the second step in writing an indirect proof?

_____

4. Find the contradiction:

a. How are the base angle measures of an isosceles triangle related?

_____

b. What must be the measure of each base angle?

_____

c. What is the sum of the angle measures in a triangle?

d. If both base angles of ΔXYZ are right angles, and the non-base angle has a measure greater than 0, what must be true of the sum of the angle measures?

_____

e. What does your assumption contradict?

_____

5. What is your conclusion?

_____

_____

5-5 Practice

Form G

Indirect Proof

Write the first step of an indirect proof of the given statement.

1. A number g is divisible by 2.

2. There are more than three red houses on the block.

3. $\square ABC$ is equilateral.

4. $m\square B < 90$

5. $\square C$ is not a right angle.

6. There are less than 15 pounds of apples in the basket.

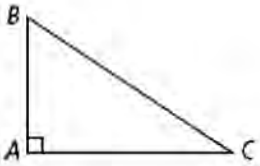
7. If the number ends in 4, then it is not divisible by 5.

8. If $\overline{MN}$ @$\overline{NO}$, then point N is on the perpendicular bisector of $\overline{MO}$.

9. If two right triangles have congruent hypotenuses and one pair of congruent legs, then the triangles are congruent.

10. If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

11. Developing Proof Fill in the blanks to prove the following statement: In right $\square ABC$, $m\square B + m\square C = 90$.



Given: right $\square ABC$

Prove: $m\square B + m\square C = 90$

Assume temporarily that $m\square B + m\square C$ _____. If $m\square B + m\square C$ _____, then $m\square A + m\square B + m\square C$ _____. According to the Triangle Angle-Sum Theorem, $m\square A + m\square B + m\square C =$ _____. This contradicts the previous statement, so the temporary assumption is _____.

Therefore, _____.

12 Use indirect reasoning to eliminate all but one of the following answers.

In what year was Barack Obama born?

A 1809 B 1909 C 1961 D 2000

5-5 Practice (continued)

Form G

Indirect Proof

Identify the two statements that contradict each other.

13. □. □ABC is acute. □□. □ABC is scalene. □□□. □ABC is equilateral.

14. □. m□B □ 90 □□. □B is acute. □□□. □B is a right angle.

15. □. $\overline{FA} \parallel \overline{AC}$
□□. $\overline{FA}$ and $\overline{AC}$ are skew.
□□□. $\overline{FA}$ and $\overline{AC}$ do not intersect.

16. □. Victoria has art class from 9:00 to 10:00 on Mondays.
□□. Victoria has math class from 10:30 to 11:30 on Mondays.
□□□. Victoria has math class from 9:00 to 10:00 on Mondays.

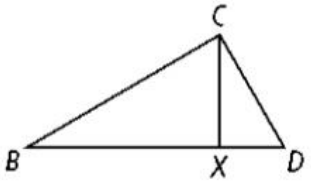
17. □. □MNO is acute.
□□. The centroid and the orthocenter for □MNO are at different points.
□□□. □MNO is equilateral.

18. □. □ABC such that □A is obtuse.
□□. □ABC such that □B is obtuse.
□□□. □ABC such that □C is acute.

19. □. The orthocenter for □ABC is outside the triangle.
□□. The median for □ABC is inside the triangle.
□□□. □ABC is an acute triangle.

Write an indirect proof.

20. Given: m□$XCD = 30$, m□$BCX = 60$, □XCD □ □XBC
Prove: $\overline{CX}$ ^ $\overline{BD}$



21. It is raining outside. Show that the temperature must be greater than 32°F.

5-5 Practice

Form K

Indirect Proof

Complete the first step of an indirect proof of the given statement.

1. There are fewer than 11 pencils in the box.
   Assume temporarily that there are ___?___ pencils in the box.

2. If a number ends in 0, then it is not divisible by 3.
   Assume temporarily that a number that ends in 0 ___?___.

3. $4x + 3 > 12$
   Assume temporarily that $4x + 3$ ☐ 12.

4. ΔRST is not an isosceles triangle.
   Assume temporarily that ___?___.

Write the first step of an indirect proof of the given statement.

5. There are more than 20 apples in a box.

6. If a number ends in x, then it is a multiple of 5.

7. $m\square XYZ < 100$

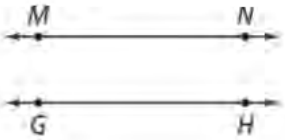
8. ΔDEF is a right triangle.

Identify the two statements that contradict each other.

9. I. $\overleftrightarrow{MN} \parallel \overleftrightarrow{GH}$

   II. $\overleftrightarrow{MN}$ and $\overleftrightarrow{GH}$ do not intersect.

   III. $\overleftrightarrow{MN}$ and $\overleftrightarrow{GH}$ are skew.



To start, identify two conditions that cannot be true at the same time.

___?___ lines must be in the same plane.
___?___ lines must not be in the same plane.
Therefore, two lines cannot be both ___?___ and ___?___.

5-5 Practice (continued)

Form K

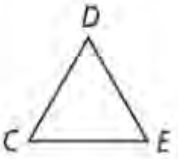
Indirect Proof

Identify the two statements that contradict each other.

10. I. ΔCDE is equilateral.

II. $m\square C$ and $m\square E$ have the same measure.

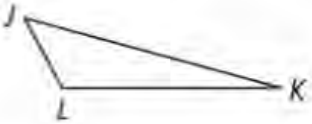
III. $m\square C > 60$



11. I. ΔJKL is scalene.

II. ΔJKL is obtuse.

III. ΔJKL is isosceles.



12. I. The orthocenter of ΔCDE is point G.

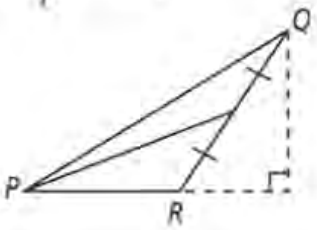
II. The centroid and orthocenter of ΔCDE are both point G.

III. ΔCDE is scalene.

13. I. The altitude of ΔPQR is outside the triangle.

II. ΔPQR is acute.

III. The median of ΔPQR is inside the triangle.



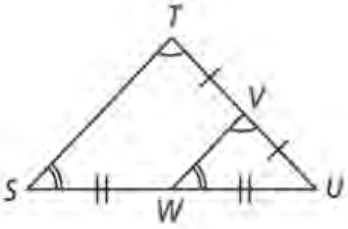
Complete the indirect proof.

14. Given: $\square S \ \square W$

$\square T \ \square$

$\square V$

Prove: $\overrightarrow{TS} \parallel \overrightarrow{VW}$



Assume temporarily that ___?___.

Then by the Converse of the ___?___, $\square S$ and $\square W$ cannot be ___?___.

This contradicts the given information that ___?___.

Therefore, $\overrightarrow{TS}$ must be ___?___ $\overrightarrow{VW}$.

5-5 Enrichment

Indirect Proof

Proofs About Triangles

Indirect reasoning is a useful way to prove things. Everyone uses indirect reasoning, sometimes without realizing it.

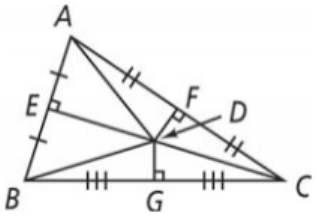
For instance, on the way to play baseball you feel a drop of water land on you, even though you had thought it was not raining. Now, however, you decide that it must be raining, even though the daylight seems too bright for such weather. You look up at the sky and see that there are no clouds, leading you to reason that it is not raining after all, and you will be able to play baseball.

You have just used the three steps of indirect reasoning. First you assume the opposite of what you want to prove. Next, you show that your assumption leads to a contradiction. Last of all, you conclude that your assumption must have been false and what you wanted to prove is true.

Use indirect reasoning to complete the following proofs of statements that have been proven using direct reasoning earlier in the chapter.

1. **Given:** ΔABC, with perpendicular bisectors meeting at point D

   Prove: $AD = BD$



2. **Given:** The incenter and circumcenter of ΔJKL are different points.

   Prove: ΔJKL is not equilateral.

5-6 Additional Vocabulary Support

Inequalities in One Triangle

The column on the left shows the steps used to solve an inequality using the Triangle Inequality Theorem. Use the column on the left to answer each question in the column on the right.

Problem **Triangle Inequality Theorem** **A handyman wants to make a fence around a garden in the shape of a triangle. He plans to use a 6-ft-long piece of fencing and a 7-ft-long piece of fencing. How long could the third piece of fencing be?**	**1.** Read the example. What do you need to find to solve the problem? _____
Choose a variable to represent the length of the third piece of fencing: x.	**2.** What is a *variable?* _____ **3.** What does *represent* mean? _____
Set up three inequalities using the Triangle Inequality Theorem: $6 + 7 > x$ $x + 6 > 7$ $x + 7 > 6$	**4.** What is the Triangle Inequality Theorem? _____ _____
Solve the inequalities: $13 > x$ $x > 1$ $x > -1$	**5.** What does it mean to *solve an inequality?* _____ _____
Form a conclusion: The value of x must be less than 13 and greater than 1. So, the length of the third piece of fencing must be greater than 1 ft and less than 13 ft.	**6.** Why is there more than one possible value of x? _____ _____ _____

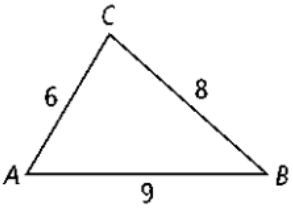
5-6 Reteaching

Inequalities in One Triangle

For any triangle, if two sides are not congruent, then the larger angle is opposite the longer side (Theorem 5-10). Conversely, if two angles are not congruent, then the longer side is opposite the larger angle (Theorem 5-11).

Problem

Use the triangle inequality theorems to answer the questions.

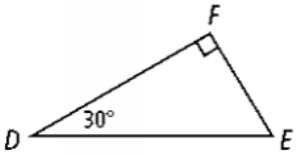


a. Which is the largest angle of ΔABC?

$\overline{AB}$ is the longest side of ΔABC. $\square C$ lies opposite $\overline{AB}$

$\square C$ is the largest angle of ΔABC.

b. What is $m\square E$? Which is the shortest side of ΔDEF?



$m\square D + m\square E + m\square F = 180$	Triangle Angle-Sum Theorem
$30 + m\square E + 90 = 180$	Substitution
$120 + m\square E = 180$	Addition
$m\square E = 60$	Subtraction Property of Equality

$\square D$ is the smallest angle of ΔDEF. Because $\overline{FE}$ lies opposite $\square D$, $\overline{FE}$ is the shortest side of ΔDEF.

Exercises

1. Draw three triangles, one obtuse, one acute, and one right. Label the vertices. Exchange your triangles with a partner.

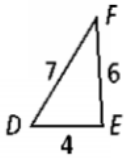
a. Identify the longest and shortest sides of each triangle.

b. Identify the largest and smallest angles of each triangle.

c. Describe the relationship between the longest and shortest sides and the largest and smallest angles for each of your partner's triangles.

Which are the largest and smallest angles of each triangle?

2.



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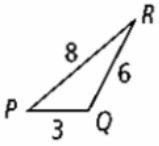

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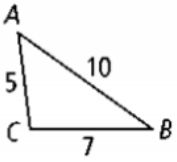
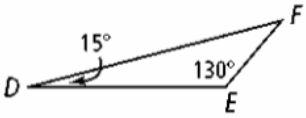


Which are the longest and shortest sides of each triangle?

5.



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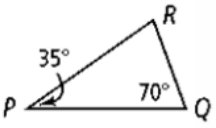
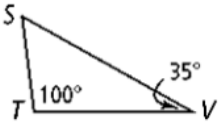


7.



5-6 Reteaching (continued)

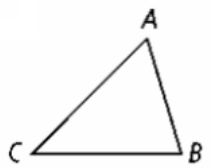
Inequalities in One Triangle

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side. This is the *Triangle Inequality Theorem.*

$AB + BC > AC$

$AC + BC > AB$

$AB + AC > BC$



Problem

A. Can a triangle have side lengths 22, 33, and 25?

Compare the sum of two side lengths with the third side length.

$22 + 33 > 25$ $\quad$ $22 + 25 > 33$ $\quad$ $25 + 33 > 22$

A triangle *can* have these side lengths.

B. Can a triangle have side lengths 3, 7, and 11?

Compare the sum of two side lengths with the third side length.

$3 + 7 < 11$ $\quad$ $3 + 11 > 7$ $\quad$ $11 + 7 > 3$

A triangle *cannot* have these side lengths.

C. Two sides of a triangle are 11 and 12 ft long. What could be the length of the third side?

Set up inequalities using x to represent the length of the third side.

$x + 11 > 12$ $\quad$ $x + 12 > 11$ $\quad$ $11 + 12 > x$

$x > 1$ $\quad$ $x > -1$ $\quad$ $23 > x$

The side length can be any value between 1 and 23 ft long.

Exercises

8. Can a triangle have side lengths 2, 3, and 7?

9. Can a triangle have side lengths 12, 13, and 7?

10. Can a triangle have side lengths 6, 8, and 9?

11. Two sides of a triangle are 5 cm and 3 cm. What could be the length of the third side?

12. Two sides of a triangle are 15 ft and 12 ft. What could be the length of the third side?

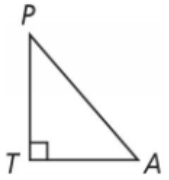
5-6 Think About a Plan

Inequalities in One Triangle

Prove this corollary to Theorem 5-11: The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Given: $\overline{PT} \perp \overline{TA}$

Prove: $PA > PT$



1. What is $m\angle T$? Explain how you know this.

_____

2. What is $m\angle P + m\angle A + m\angle T$? Explain how you know this.

_____

3. What is $m\angle P + m\angle A$? Explain how you know this.

_____

_____

4. Write an inequality to show $m\angle A$.

5. Write an inequality to show the relationship between $m\angle A$ and $m\angle T$.

6. Which side lies opposite $\angle A$ and which side lies opposite $\angle T$?

_____

7. What is Theorem 5-11?

_____

_____

8. What can you conclude about PA and PT?

_____

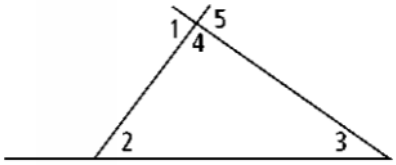
5-6 Practice

Form G

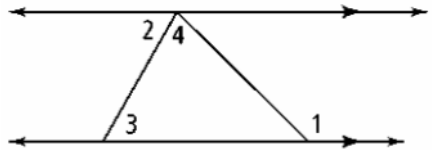
Inequalities in One Triangle

Explain why $m\angle 1 > m\angle 2$.

1.

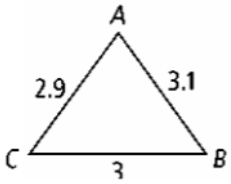


2.



For Exercises 3–6, list the angles of each triangle in order from smallest to largest.

3.



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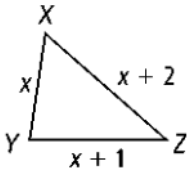

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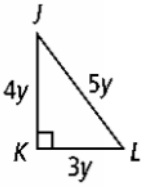

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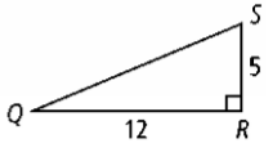
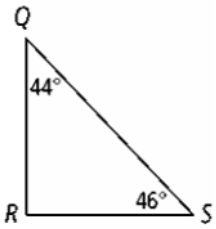


For Exercises 7–10, list the sides of each triangle in order from shortest to longest.

7.



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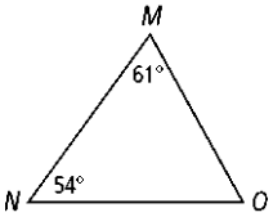


9. ΔABC, with $m\angle A = 99$, $m\angle B = 44$, and $m\angle C = 37$

10. ΔABC, with $m\angle A = 122$, $m\angle B = 22$, and $m\angle C = 36$

For Exercises 11 and 12, list the angles of each triangle in order from smallest to largest.

11. ΔABC, where $AB = 17$, $AC = 13$, and $BC = 29$

12. ΔMNO, where $MN = 4$, $NO = 12$, and $MO = 10$

5-6 Practice (Continued)

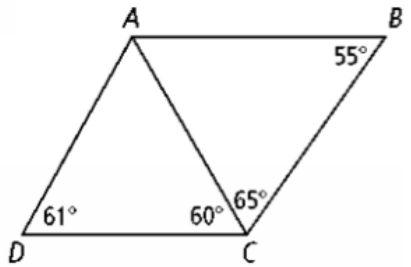
Form G

Inequalities in One Triangle

Form G

Determine which side is shortest in the diagram.

13.



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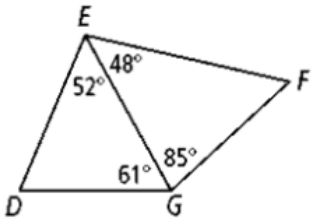


Can a triangle have sides with the given lengths? Explain.

15. 8 cm, 7 cm, 9 cm

16. 7 ft, 13 ft, 6 ft

17. 20 in., 18 in., 16 in.

18. 3 m, 11 m, 7 m

Algebra The lengths of two sides of a triangle are given. Describe the possible lengths for the third side.

19. 5, 11

20. 12, 12

21. 25, 10

22. 6, 8

23. Algebra List the sides in order from shortest to longest in ΔPQR, with $m\square P = 45$, $m\square Q = 10x + 30$, and $m\square R = 5x$.

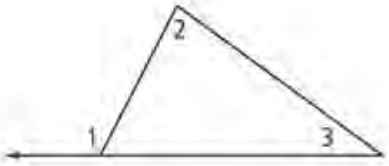
24. Algebra List the sides in order from shortest to longest in ΔABC, with $m\square A = 80$, $m\square B = 3x + 5$, and $m\square C = 5x \quad 1$.

25. Error Analysis A student draws a triangle with a perimeter 36 cm. The student says that the longest side measures 18 cm. How do you know that the student is incorrect? Explain.

5-6 Practice

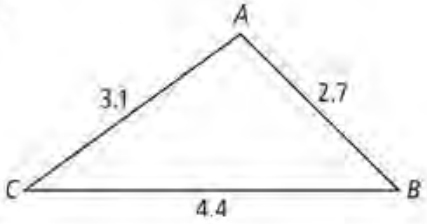
Form K

1. Explain the relationship of $m\angle 1$, $m\angle 2$, and $m\angle 3$.
The measure of an exterior angle of a triangle is ___?___ than the measure of each of its remote ___?___ angles.
$\angle 1$ is an ___?___ angle of the triangle, so $m\angle 1 >$ ___?___ and $m\angle 1 >$ ___?___



For Exercises 2–5, list the angles of each triangle in order from smallest to largest.

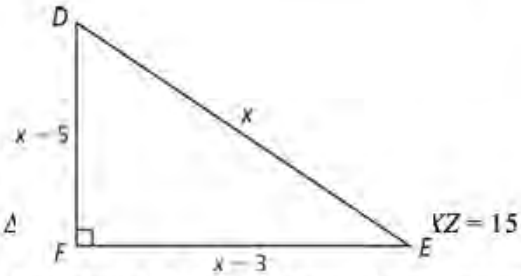
2.



To start, order the side lengths from least to greatest.

☐ < ☐ < ☐

3.



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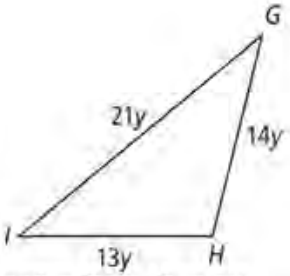
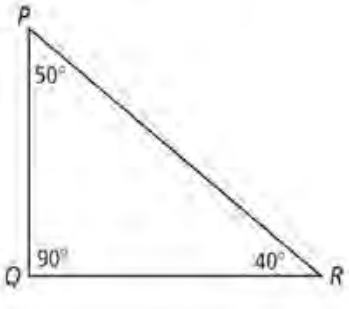


5. Δ $YZ = 15$

For Exercises 6–8, list the sides of each triangle in order from shortest to longest.

6.



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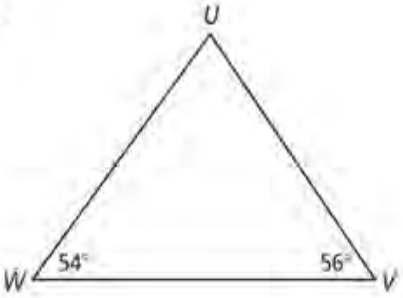


8. ΔMNO, where $m\angle M = 56$, $m\angle N = 108$, and $m\angle O = 16$

9. Algebra List the sides in order from shortest to longest in ΔXYZ, with $m\angle X = 50$, $m\angle Y = 5x + 10$, and $m\angle Z = 5x$.

5-6 Practice (continued)

Form K

Inequalities in One Triangle

Can a triangle have sides with the given lengths? Explain.

10. 10 in., 13 in., 18 in.

To start, choose two sides and see if their sum exceeds the third side.

$10 + 13$ [$\overset{?}{>}$] 18 yes / no (Circle the correct answer.)

Check the other two sums.

11. 6 m, 5 m, 12 m

12. 11 ft, 8 ft, 18 ft

Algebra The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side.

13. 4, 8

To start, write the inequalities relating the known side lengths and the unknown side length.

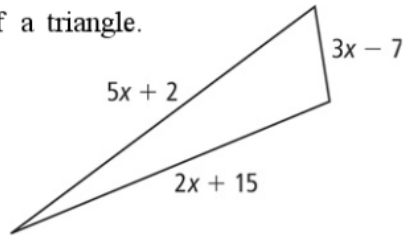
$x + 4 > 8$ $\qquad$ $x + 8 > \square$ $\qquad$ $8 + 4 > \square$

14. 13, 8

15. 10, 15

16. Error Analysis A student draws a triangle with a perimeter of 12 in. The student says that the longest side measures 7 in. How do you know that the student is incorrect? Explain.

17. Algebra ΔXYZ has the side lengths shown at the right. What values of x result in side lengths that could be the sides of a triangle. (*Hint:* Write and solve three inequalities.)



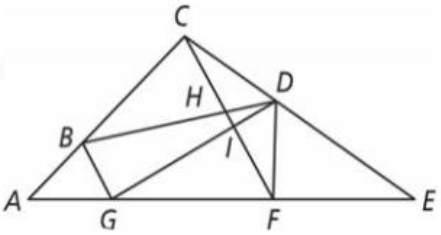
5-6 Enrichment

Inequalities in One Triangle

The First of the Greek Geometers

Euclid organized the known geometry of his day into one of history's all-time best sellers, *The Elements*. However, Euclid was not the father of Greek geometry. That honor belongs to a philosopher known as one of the Seven Wise Men of ancient Greece, who first introduced geometry to Greece. He founded the geometry of lines and was probably the first great mathematician in history.

To discover the identity of this great mathematician and philosopher, consider this diagram in which every angle that appears to be acute is acute, and every angle that appears to be obtuse is obtuse. Answer each question individually, and without the use of a ruler or a protractor.



Each statement below is true (T), false (F), or undecidable (U). The answers to Exercises 1–3 will supply the hint to decode the first letter of the name, the answers to Exercises 4–6 to the second letter, and so on.

A – TTT	B – TTF	C – TTU	D – TFT	E – TFF	F – TFU	G – TUT
H – TUF	I – TUU	J – FTT	K – FTF	L – FTU	M – FFT	N – FFF
O – FFU	P – FUT	Q – FUF	R – FUU	S – UTT	T – UTF	U – UTU
V – UFT	W – UFF	X – UFU	Y – UUT	Z – UUF		

Relation	T, U, or F	Relation	T, U, or F
1. $BG > DF$	?	**10.** $m\angle AGB > m\angle AGD$	?
2. $CF > DF$	?	**11.** $m\angle CBG > m\angle BAG$	?
3. $m\angle CEF > m\angle CFE$	?	**12.** $AG > CH$	?
4. $m\angle BHC > m\angle CDH$	?	**13.** $CF > HI$	?
5. $DG > DE$	?	**14.** $m\angle CFD > m\angle AFD$	?
6. $m\angle DGF > m\angle DFE$	?	**15.** $BH > BD$	?
7. $DG > IG$	?	**16.** $BH > EF$	?
8. $m\angle AFD > m\angle AEC$	?	**17.** $CE > CF$	?
9. $m\angle CDF > m\angle GBD$	?	**18.** $m\angle ACE > m\angle ACF$	?

The name of the mathematician is ___ ___ ___ ___ ___ ___ .

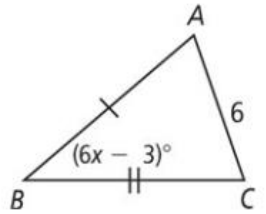
19. List at least five reasons that you used to determine whether the statements above were true, false, or undecidable.

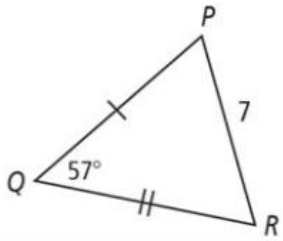
5-7 Additional Vocabulary Support

Inequalities in Two Triangles

You want to use the Converse of the Hinge Theorem to find the possible values for x in the triangle at the right.

You wrote these steps to solve the problem on the note cards, but they got mixed up.





Solve the first inequality:

$6x < 60$, so

$x < 10$.

Write two inequalities:

$6x - 3 < 57$ and

$6x - 3 > 0$.

Solve the second inequality:

$6x > 3$, so

$x > 0.5$.

Compare measures of the included angles.

$PR > AC$, so

$m\angle Q > m\angle B$.

Think: The triangles ABC and PQR have two pairs of congruent sides. Identify the included angles: $\angle B$ and $\angle Q$.

Write an inequality for the possible values of x:

$0.5 < x < 10$.

Use the note cards to write the steps in order.

1. First, _____

_____

2. Second, _____

3. Third, _____

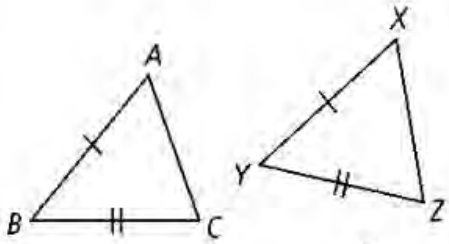
4. Fourth, _____

5. Next, _____

6. Then, _____

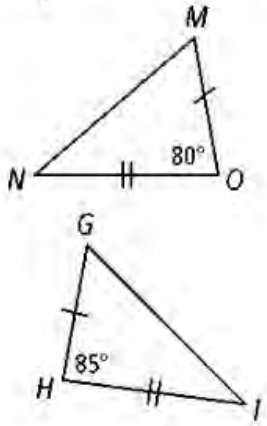
5-7 Reteaching

Consider ΔABC and Δ*XYZ*. If $\overline{AB} \cong \overline{XY}, \overline{BC} \cong \overline{YZ}$, and $m\angle Y > m\angle B$, then $XZ > AC$. This is the *Hinge Theorem* (SAS Inequality Theorem).



Problem

Which length is greater, *GI* or *MN*?



Identify congruent sides: $\overline{MO} \cong \overline{GH}$ and $\overline{NO} \cong \overline{HI}$.

Compare included angles: $m\angle H > m\angle O$.

By the Hinge Theorem, the side opposite the larger included angle is longer.

So, $GI > MN$.

Problem

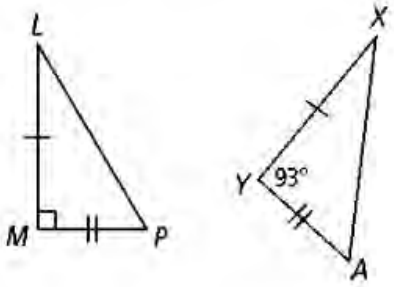
At which time is the distance between the tip of a clock's hour hand and the tip of its minute hand greater, 3:00 or 3:10?

Think of the hour hand and the minute hand as two sides of a triangle whose lengths never change, and the distance between the tips of the hands as the third side. 3:00 and 3:10 can then be represented as triangles with two pairs of congruent sides. The distance between the tips of the hands is the side of the triangle opposite the included angle.

At 3:00, the measure of the angle formed by the hour hand and minute hand is 90°. At 3:10, the measure of the angle is less than 90°.

So, the distance between the tip of the hour hand and the tip of the minute hand is greater at 3:00.

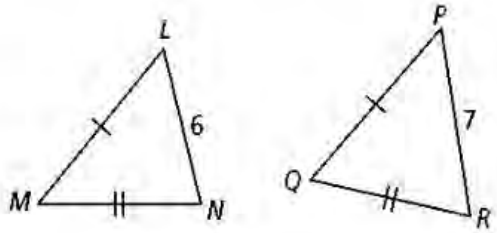
Exercises



1. What is the inequality relationship between *LP* and *XA* in the figure at the right?
2. At which time is the distance between the tip of a clock's hour hand and the tip of its minute hand greater, 5:00 or 5:15?

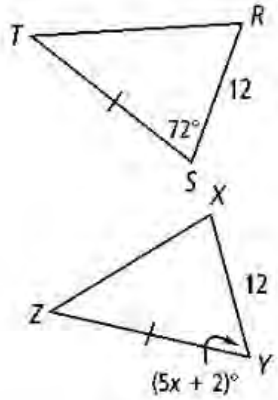
5-7 Reteaching (continued)

Consider ΔLMN and ΔPQR. If $LM \cong PQ, MN \cong QR$, and $PR > LN$, then $m\angle Q > m\angle M$. This is the *Converse of the Hinge Theorem* (SSS Inequality Theorem).



Problem

$TR > ZX$. What is the range of possible values for x?



The triangles have two pairs of congruent sides, because $RS = XY$ and $TS = ZY$. So, by the Converse of the Hinge Theorem, $m\angle S > m\angle Y$.

Write an inequality:

$72 > 5x + 2$	Converse of the Hinge Theorem
$70 > 5x$	Subtract 2 from each side.
$14 > x$	Divide each side by 5.

Write another inequality:

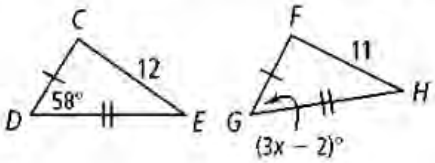
$m\angle Y > 0$	The measure of an angle of a triangle is greater than 0.
$5x + 2 > 0$	Substitute.
$5x > -2$	Subtract 2 from each side.
$x > -\frac{2}{5}$	Divide each side by 5.

So, $-\frac{2}{5} < x < 14$.

Exercises

Find the range of possible values for each variable.

3.



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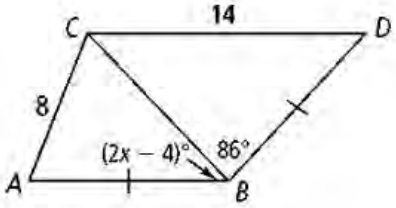

5. Reasoning An equilateral triangle has sides of length 5, and an isosceles triangle has side lengths of 5, 5, and 4. Write an inequality for x, the measure of the vertex angle of the isosceles triangle.

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5-7 Think About a Plan

Inequalities in Two Triangles

Reasoning The legs of a right isosceles triangle are congruent to the legs of an isosceles triangle with an 80° vertex angle. Which triangle has a greater perimeter? How do you know?

1. How can you use a sketch to help visualize the problem?
   Draw a sketch.

2. The triangles have two pairs of congruent sides. For the right triangle, what is the measure of the included angle? How do you know this?

   _____

3. For the second triangle, what is the measure of the included angle? How do you know this?

   _____

4. How could you find the perimeter of each triangle?

   _____

5. How does the sum of the lengths of the legs in the right triangle compare to the sum of the lengths of the legs in the other triangle?

   _____

6. Write formulas for the perimeters of each triangle. Use the variable ℓ for leg length, b_1 for base length of the right triangle, and b_2 for base length of the second triangle.

7. What values do you need to compare to find the triangle with the greater perimeter?

   _____

8. How can you use the Hinge Theorem to find which base length is longer?

   _____

9. Which base length is longer? _____

10. Which triangle has the greater perimeter? _____

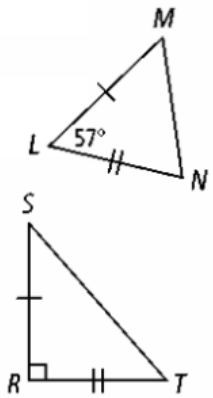
5-7 Practice

Form G

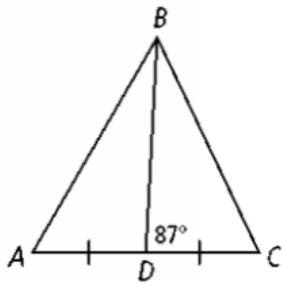
Inequalities in Two Triangles

Write an inequality relating the given side lengths. If there is not enough information to reach a conclusion, write *no conclusion.*

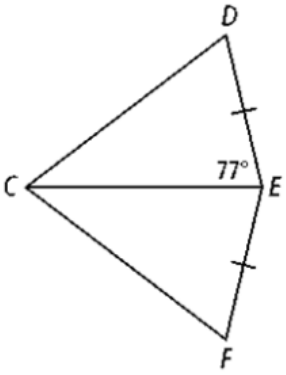
1. *ST* and *MN*



2. *BA* and *BC*



3. *CD* and *CF*

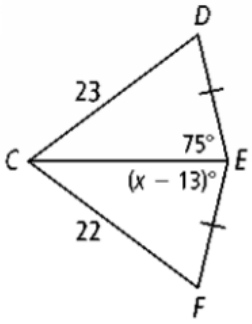


4. A crocodile opens his jaws at a 30□ angle. He closes his jaws, then opens them again at a 36□ angle. In which case is the distance between the tip of his upper jaw and the tip of his lower jaw greater? Explain.

5. At which time is the distance between the tip of the hour hand and the tip of the minute hand greater, 2:20 or 2:25?

Find the range of possible values for each variable.

6.



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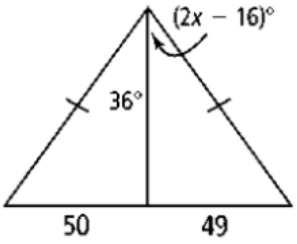

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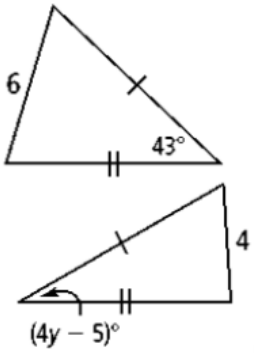
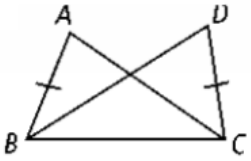


9. In the triangles at the right, $AB = DC$ and $m\angle ABC < m\angle DCB$. Explain why $AC < BD$.

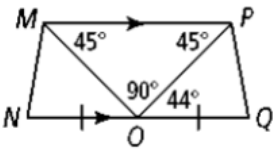


5-7 Practice (continued)

Form G

Inequalities in Two Triangles

Copy and complete with > or <. Explain your reasoning.



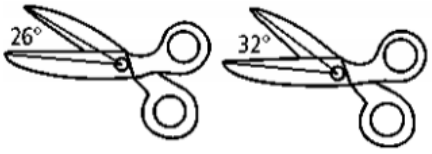
10. $m\angle POQ$ ___?___ $m\angle MON$

11. MN ___?___ PQ

12. MP ___?___ OP

13. Jogger A and Jogger B start at the same point. Jogger A travels 0.9 mi due east, then turns 120° clockwise, then travels another 3 mi. Jogger B travels 0.9 mi due west, then turns 115° counterclockwise, then travels another 3 mi. Do the joggers end in the same place? Explain.

14. In the diagram at the right, in which position are the tips of the scissors farther apart?

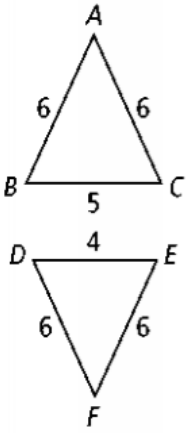


Position A **Position B**

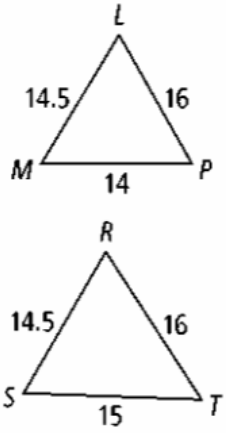
15. The legs of an isosceles triangle with a 65° vertex angle are congruent with the sides of an equilateral triangle. Which triangle has a greater perimeter? How do you know?

Write an inequality relating the given angle measures. If there is not enough information to reach a conclusion, write *no conclusion*.

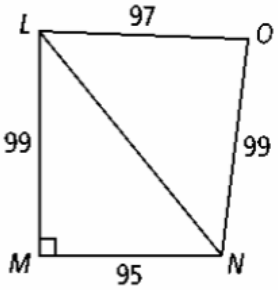
16. $m\angle A$ and $m\angle F$



17. $m\angle L$ and $m\angle R$



18. $m\angle MLN$ and $m\angle ONL$



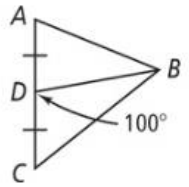
5-7 Practice

Form K

Write an inequality relating the given side lengths. If there is not enough information to reach a conclusion, write *no conclusion*.

1. *AB* and *CB*

To start, determine whether the triangles have two pairs of congruent sides.



$\overline{AD} \cong \overline{CD}$

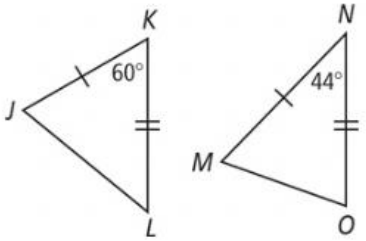
$\overline{DB} \cong$?

Then compare the hinge angles.

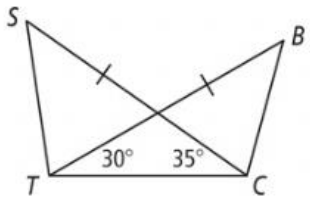
$m\angle CDB = \square$

$m\angle \square = \square$

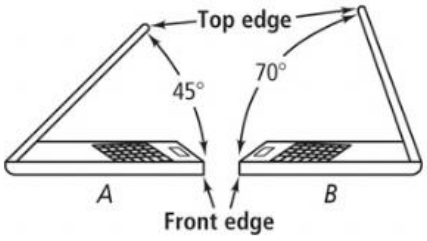
2. *JL* and *MO*



3. *ST* and *BT*

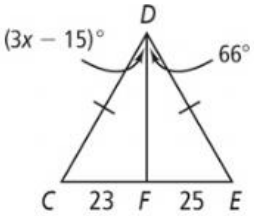


4. Two identical laptops are shown at the right. In which laptop is the distance from the top edge of the screen to the front edge of the keyboard greater? Explain.



Algebra Find the range of possible values for each variable.

5.



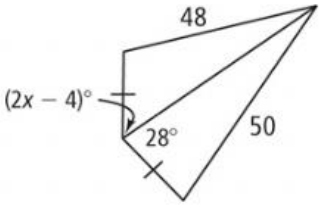
$m\angle CDF < m\angle EDF$

$x < \square$

$m\angle CDE > 0$

$x > \square$

6.

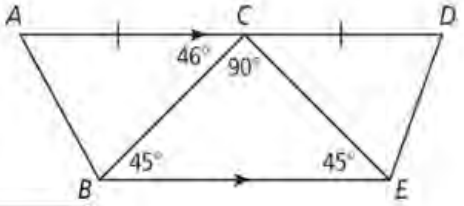


$\square < x < \square$

5-7 Practice (continued)

Form K

Use the diagram at the right for Exercises 7–9. Complete each comparison with < or >. Then complete the explanation.



7. $m\angle ACB$ ☐ $m\angle DCE$

☐ forms a straight angle with ☐ and ☐

The measure of $\angle DCE$ is ☐.

8. AB ☐ DE

Because ΔBCE is an isosceles triangle, ☐ = ☐.

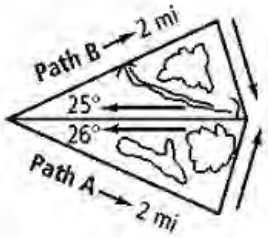
☐ and ☐ have two pairs of congruent sides.

So, by the ? Theorem, AB ☐ DE.

9. BE ☐ CE

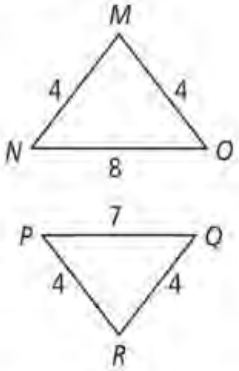
The longest side of a Δ is opposite the angle with the ? measure.

10. The diagram shows two paths that lead through a park. Would a jogger run a greater distance on Path A or Path B? Explain.



Write an inequality relating the given angle measures.

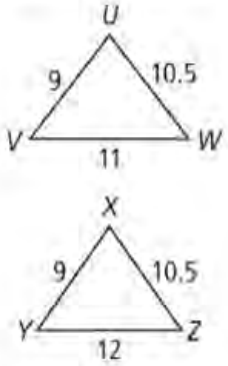
11. $m\angle M$ and $m\angle R$



$\overline{MN}$, $\overline{MO}$, $\overline{PR}$, and $\overline{QR}$ are ?.

NO ☐ PQ

12. $m\angle U$ and $m\angle X$



5-7 Enrichment

Inequalities in Two Triangles

Comparing Distances

Triangle inequalities can be used to solve real-world problems like the following:

Samantha works at a bakery, and delivers pastries to local businesses.

On Tuesdays and Thursdays, she walks due west out of the bakery and travels 150 ft to the Sewing Shop. From there, she turns 45° toward the south, and travels another 150 ft to the gas station. From the gas station she travels 107 ft due east to Howard's Flowers. Howard's Flowers is due south of the Sewing Shop.

On Mondays and Wednesdays, Samantha travels 150 ft due south of the bakery to Richard's Records. From there, she makes a 40° turn toward the west and travels 107 ft to Dualla's Hardware.

On Fridays, Samantha makes the Monday-Wednesday route, but also goes to Melki's Meats. From the hardware store, she turns 133° to her right and travels 150 ft northeast to Melki's.

1. Make a drawing of Samantha's delivery routes. Label all angles and all distances between buildings.

2. Use your drawing to answer the following questions. Assume a straight path between each set of points.

a. Which is greater, the distance between the Sewing Shop and Howard's Flowers, or the distance between Richard's Records and Melki's Meats?

b. Which is greater, the distance between the bakery and Howard's Flowers, or the distance between Howard's Flowers and the gas station?

3. One day Samantha has to make a delivery from the bakery to Richard's Records, then to Melki's Meats. What is the minimum distance she must walk to get to Melki's via Richard's? Explain.