Geometry

Learning Guide
Grade: HS  Subject: Geometry (Worksheets taken from Pearson)

<table>
<thead>
<tr>
<th>Topic: Congruent Triangles</th>
</tr>
</thead>
</table>

**What Your Student is Learning:** Recognizing congruent figures and their parts, proving triangles congruent, proving right triangles congruent

**Background and Context for Parents:** This unit builds on your students’ previous understandings and skills related to angles and triangles. Some of the big ideas in this unit that they explore are

- How do you identify corresponding parts of congruent *(identical in form)* triangles?
  - Comparing the corresponding *(matching)* parts of two figures can show whether figures are congruent.
  - We use tick marks and angle marks to label corresponding sides and corresponding angles (see pic).
- How do you show that two triangles are congruent?
  - Two triangles can be shown to be congruent without having to show that all corresponding parts are congruent. Triangles can be proven to be congruent by 1) three pairs of corresponding sides (SSS)  2) two pairs of corresponding sides and one pair of corresponding angles (SAS) 3) one pair of corresponding sides and two pair of corresponding angles (ASA) (AAS), or one pair of right angles, a pair of legs, and a pair of hypotenuses.
  - Note: Side-Side-Angle (SSA) and Angle-Angle-Angle (AAA) do NOT work
- How can you tell whether a triangle is isoceles (two equal sides) or equilateral (three equal sides)?
  - The angles and sides of isoceles and equilateral triangles have special relationships.
  - Look for the number of congruent sides and angles.

**Ways to support your student:**

- Read the problem outloud to them.
- Before giving your student the answer to their question or specific help, ask them “What have you tried so far?, What do you know?, What might be a next step?"
- After your student has solved it, and before you tell them it’s correct or not, have them explain to you how they got their solution and if they think their answer makes sense.
- Grab some straws, paper clips, and yarn and do this activity with your student to help them discover congruence [https://www.mathgiraffe.com/blog/discovery-congruent-triangles](https://www.mathgiraffe.com/blog/discovery-congruent-triangles)

**Online Resources for Students:**
- Desmos: [https://tinyurl.com/yac8mmqn](https://tinyurl.com/yac8mmqn) - Investigating congruent triangles
- Proving Two Triangles Congruent: [https://www.youtube.com/watch?v=eWfEXKmNCKy](https://www.youtube.com/watch?v=eWfEXKmNCKy)
- Geogebra Applet: [https://www.geogebra.org/m/dVUczkUm#material/nEqHyhVN](https://www.geogebra.org/m/dVUczkUm#material/nEqHyhVN)
# Additional Vocabulary Support

## Congruent Figures

### Concept List

<table>
<thead>
<tr>
<th>algebraic equation</th>
<th>angle measure</th>
<th>congruency statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>congruent angles</td>
<td>congruent polygons</td>
<td>proof</td>
</tr>
<tr>
<td>congruent triangles</td>
<td>segment measure</td>
<td></td>
</tr>
</tbody>
</table>

Choose the concept from the list above that best represents the item in each box.

1. \(GH = ST\)

2. \(m \angle A = 45\)

3. [Diagram of congruent figures]

4. \(YZ = MN\)

5. \(\triangle ABC \cong \triangle XYZ\)

6. Given: \(BD\) is the angle bisector of \(\angle ABC\), and \(BD\) is the perpendicular bisector of \(AC\).
   
   Prove: \(\triangle ADB \cong \triangle CDB\)

7. \(m \angle H = 5x\)  
   \(m \angle W = x + 28\)

   Solve \(5x = x + 28\) to find the measures of \(\angle H\) and \(\angle W\).

8. \(BC = 3\) cm

9. \(\angle ADB\) and \(\angle SDT\) are vertical angles. So, \(\angle ADB \cong \angle SDT\).
Given $ABCD \cong QRST$, find corresponding parts using the names. Order matters.

For example, $\overline{ABCD}$  
This shows that $\angle A$ corresponds to $\angle Q$.
Therefore, $\angle A \cong \angle Q$.

For example, $\overline{BC}$  
This shows that $\overline{BC}$ corresponds to $\overline{RS}$.
Therefore, $\overline{BC} \cong \overline{RS}$.

**Exercises**

Find corresponding parts using the order of the letters in the names.

1. Identify the remaining three pairs of corresponding angles and sides between $ABCD$ and $QRST$ using the circle technique shown above.

   Angles: $ABCD$  
   $QRST$

   Sides: $ABCD$  
   $QRST$

2. Which pair of corresponding sides is hardest to identify using this technique?

Find corresponding parts by redrawing figures.

3. The two congruent figures below at the left have been redrawn at the right. Why are the corresponding parts easier to identify in the drawing at the right?

4. Redraw the congruent polygons at the right in the same orientation. Identify all pairs of corresponding sides and angles.

5. $MNOP \cong QRST$. Identify all pairs of congruent sides and angles.
Problem

Given $\triangle ABC \cong \triangle DEF$, $m \angle A = 30$, and $m \angle E = 65$, what is $m \angle C$?

How might you solve this problem? Sketch both triangles, and put all the information on both diagrams.

$m \angle A = 30$; therefore, $m \angle D = 30$. How do you know?

Because $\angle A$ and $\angle D$ are corresponding parts of congruent triangles.

Exercises

Work through the exercises below to solve the problem above.

6. What angle in $\triangle ABC$ has the same measure as $\angle E$? What is the measure of that angle? Add the information to your sketch of $\triangle ABC$.

7. You know the measures of two angles in $\triangle ABC$. How can you find the measure of the third angle?

8. What is $m \angle C$? How did you find your answer?

Before writing a proof, add the information implied by each given statement to your sketch. Then use your sketch to help you with Exercises 9–12.

Add the information implied by each given statement.

9. Given: $\angle A$ and $\angle C$ are right angles.

10. Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$.

11. Given: $\angle ADB \cong \angle CBD$.

12. Can you conclude that $\angle ABD \cong \angle CDB$ using the given information above? If so, how?

13. How can you conclude that the third side of both triangles is congruent?
4-1 Think About a Plan
Congruent Figures

Algebra Find the values of the variables.

Know

1. What do you know about the measure of each of the non-right angles?

2. What do you know about the length of each of the legs?

3. What types of triangles are shown in the figure?

Need

4. What information do you need to know to find the value of $x$?

5. What information do you need to know to find the value of $r$?

Plan

6. How can you find the value of $x$? What is its value?

7. How do you find the value of $r$? What is its value?
4-2

Additional Vocabulary Support
Triangle Congruence by SSS and SAS

Problem
Use the figure at the right. How can you prove that \( \triangle ABC \cong \triangle XYZ \)? Justify each step.

Given: The figure at the right

Prove: \( \triangle ABC \cong \triangle XYZ \)

1) \( \overline{AB} \cong \overline{XY} \)  
2) \( \overline{BC} \cong \overline{YZ} \)  
3) \( \angle A \cong \angle X \)  
4) \( \angle C \cong \angle Z \)  
5) \( \angle B \cong \angle Y \)  
6) \( \triangle ABC \cong \triangle XYZ \)  

Side-Angle-Side (SAS) Postulate

Exercises

1. Use the figure at the right. How can you prove that \( \triangle GMH \cong \triangle TMS \)? Justify each step.

Given: \( M \) is the midpoint of \( \overline{HS} \) and \( \overline{GT} \).

Prove: \( \triangle GMH \cong \triangle TMS \)

1) \( M \) is the midpoint of \( \overline{HS} \) and \( \overline{GT} \).
2) \( \overline{GM} \cong \overline{TM} \)
3) \( \overline{HM} \cong \overline{SM} \)
4) \( \angle GMH \cong \angle TMS \)
5) \( \angle GMH \cong \angle TMS \)

2. Use the figure at the right. How can you prove that \( \triangle GHI \cong \triangle JHI \)? Justify each step.

Given: \( H \) is the midpoint of \( \overline{GJ} \).

Prove: \( \triangle GHI \cong \triangle JHI \)

1) \( H \) is the midpoint of \( \overline{GJ} \).
2) \( \overline{GH} \cong \overline{JH} \)
3) \( \overline{HI} \cong \overline{HI} \)
4) \( \overline{GI} \cong \overline{JI} \)
5) \( \angle GHI \cong \angle JHI \)
You can prove that triangles are congruent using the two postulates below.

**Postulate 4-1: Side-Side-Side (SSS) Postulate**
If all three sides of a triangle are congruent to all three sides of another triangle, then those two triangles are congruent.

If \( JK \cong XY \), \( KL \cong YZ \), and \( JL \cong XZ \), then \( \triangle JKL \cong \triangle XYZ \).

In a triangle, the angle formed by any two sides is called the *included angle* for those sides.

**Postulate 4-2: Side-Angle-Side (SAS) Postulate**
If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then those two triangles are congruent.

If \( P Q \cong D E \), \( P R \cong D F \), and \( \angle P \cong \angle D \), then \( \triangle PQR \cong \triangle DEF \).

\( \angle P \) is included by \( P Q \) and \( PR \). \( \angle D \) is included by \( ED \) and \( DF \).

**Exercises**

1. What other information do you need to prove \( \triangle TRF \cong \triangle DFR \) by SAS? Explain.

2. What other information do you need to prove \( \triangle ABC \cong \triangle DEF \) by SAS? Explain.

3. **Developing Proof** Copy and complete the flow proof.
   Given: \( DA \cong MA \), \( AJ \cong AZ \)
   Prove: \( \triangle JDA \cong \triangle ZMA \)

   ![Flow Proof Diagram](image)
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write not enough information. Explain your answer.

4. 

5. 

6. 

7. Given: \( PO \cong SO \), \( O \) is the midpoint of \( NT \). 
Prove: \( \Delta NOP \cong \Delta TOS \)

8. Given: \( HI \cong HG \), \( FH \perp GI \)
Prove: \( \Delta FHI \cong \Delta FHG \)

9. A carpenter is building a support for a bird feeder. He wants the triangles on either side of the vertical post to be congruent. He measures and finds that \( AB \cong DE \) and that \( AC \cong DF \). What would he need to measure to prove that the triangles are congruent using SAS? What would he need to measure to prove that they are congruent using SSS?

10. An artist is drawing two triangles. She draws each so that two sides are 4 in. and 5 in. long and an angle is 55°. Are her triangles congruent? Explain.
Think About a Plan
Triangle Congruence by SSS and SAS

Use the Distance Formula to determine whether $\triangle ABC$ and $\triangle DEF$ are congruent. Justify your answer.
$A(1, 4), B(5, 5), C(2, 2)$
$D(-5, 1), E(-1, 0), F(-4, 3)$

Understanding the Problem
1. You need to determine if $\triangle ABC \cong \triangle DEF$. What are the three ways you know to prove triangles congruent?

2. What information is given in the problem?

Planning the Solution
3. If you use the SSS Postulate to determine whether the triangles are congruent, what information do you need to find?

4. How can you find distances on a coordinate plane without measuring?

5. In an ordered pair, which number is the $x$-coordinate? Which is the $y$-coordinate?

Getting an Answer
6. Find the length of each segment using the Distance Formula,
   $$D = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}.$$ Your answers may be in simplest radical form.
   $\overline{AB}$
   $\overline{BC}$
   $\overline{CA}$
   $\overline{DE}$
   $\overline{EF}$
   $\overline{FD}$

7. Using the SSS Postulate, are the triangles congruent? Explain.
4-3
Additional Vocabulary Support
Triangle Congruence by ASA and AAS

Problem
Given: The figure at the right
Prove: $\triangle ABQ \cong \triangle XYQ$

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, list the information that is given directly in the diagram.</td>
<td>$BQ = YQ$ $\angle A$ and $\angle X$ are right $\triangle$.</td>
<td>Given</td>
</tr>
<tr>
<td>Second, use the fact that all right angles are congruent to each other.</td>
<td>$\angle A \cong \angle X$</td>
<td>All right angles are congruent.</td>
</tr>
<tr>
<td>Next, use the fact that vertical angles are congruent.</td>
<td>$\angle AQB \cong \angle XQY$</td>
<td>Vertical Angles Theorem</td>
</tr>
<tr>
<td>Finally, determine which theorem can be used to prove the triangles congruent using the information listed above.</td>
<td>$\triangle ABQ \cong \triangle XYQ$</td>
<td>Angle-Angle-Side (AAS) Theorem</td>
</tr>
</tbody>
</table>

Exercise
Given: The figure at the right
Prove: $\triangle HJL \cong \triangle KJL$

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IJ \parallel KL$ $HL \parallel KJ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\angle HJL \cong \angle KJL$ $\angle HLJ \cong \angle KJL$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$JL \cong LJ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\triangle HJL \cong \triangle KJL$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4-3 Reteaching

**Problem**
Can the ASA Postulate or the AAS Theorem be applied directly to prove the triangles congruent?

a. Because $\angle RDE$ and $\angle ADE$ are right angles, they are congruent. $ED \cong ED$ by the Reflexive Property of $\equiv$, and it is given that $\angle R \cong \angle A$. Therefore, $\triangle RDE \cong \triangle ADE$ by the AAS Theorem.

b. It is given that $CH \cong FH$ and $\angle F = \angle C$. Because $\angle CHE$ and $\angle FHB$ are vertical angles, they are congruent. Therefore, $\triangle CHE \cong \triangle FHB$ by the ASA Postulate.

**Exercises**
Indicate congruences.

1. Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.

2. Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.

3. Draw and mark two triangles that are congruent by either the ASA Postulate or the AAS Theorem.

**What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?**

4. ASA Postulate  
5. AAS Theorem  
6. ASA Postulate

7. AAS Theorem  
8. AAS Theorem  
9. ASA Postulate
0. Provide the reason for each step in the two-column proof:
   Given: \(TX \parallel VW\), \(TU = VU\), \(\angle XTU \cong \angle WVU\), \(\angle UWV\) is a right angle.
   Prove: \(\triangle TUX \cong \triangle VUW\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (\angle UWV) is a right angle.</td>
<td>1) (_)</td>
</tr>
<tr>
<td>2) (_)</td>
<td>2) (_)</td>
</tr>
<tr>
<td>3) (_)</td>
<td>3) (_)</td>
</tr>
<tr>
<td>4) (_)</td>
<td>4) (_)</td>
</tr>
<tr>
<td>5) (\angle UXT) is a right angle.</td>
<td>5) (_)</td>
</tr>
<tr>
<td>6) (\angle UWV \cong \angle UXT)</td>
<td>6) (_)</td>
</tr>
<tr>
<td>7) (_)</td>
<td>7) (_)</td>
</tr>
<tr>
<td>8) (\angle XTU = \angle WVU)</td>
<td>8) (_)</td>
</tr>
<tr>
<td>9) (\triangle TUX \cong \triangle VUW)</td>
<td>9) (_)</td>
</tr>
</tbody>
</table>

1. Write a paragraph proof.
   Given: \(WX \parallel ZY; \parallel WZ \parallel XY\)
   Prove: \(\triangle WXY \cong \triangle YZW\)

2. Developing Proof Complete the proof by filling in the blanks.
   Given: \(\angle A \cong \angle C\), \(\angle 1 \cong \angle 2\)
   Prove: \(\triangle ABD \cong \triangle CDB\)
   Proof: \(\angle A \cong \angle C\) and \(\angle 1 \cong \angle 2\) are given. \(\overline{DB} = \overline{BD}\) by \(\_\) 
   So, \(\triangle ABD \cong \triangle CDB\) by \(\_\)

3. Write a paragraph proof.
   Given: \(\angle 1 \cong \angle 6\), \(\angle 3 \cong \angle 4\), \(LP \parallel OP\)
   Prove: \(\triangle LMP \cong \triangle ONP\)
Think About a Plan
Triangle Congruence by ASA and AAS

Given: \( AB \parallel CD, \ AD \parallel CB \)

Prove: \( \triangle ABC \cong \triangle CDA \)

1. What do you need to find to solve the problem?

2. What are the corresponding parts of the two triangles?

3. What word would you use to describe \( AC \)?

4. What can you show about angles in the triangles that can indicate congruency?

5. What do you know about a side or sides of the triangles that can be used to show congruency?

6. Write a proof in paragraph form.
There are two sets of note cards below that show how to prove $BD$ is the perpendicular bisector of $AE$. The set on the left has the statements and the set on the right has the reasons. Write the statements and the reasons in the correct order.

**Statements**

1. $\angle BAC = \angle DEC$
2. $AC \equiv EC$
3. $\triangle ACB = \triangle ECD$
4. $BD$ is the perpendicular bisector of $AE$.  
5. $BC \equiv DC$; $\angle ACB$ and $\angle ECD$ are right angles; $AB \parallel DE$
6. $\angle ACB = \angle ECD$

**Reasons**

1. Definition of the perpendicular bisector
2. Angle-Angle-Side (AAS) Theorem
3. When parallel lines are cut by a transversal, alternate interior angles are congruent.
4. Corresponding parts of congruent triangles are congruent.
5. Given
6. All right angles are congruent.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>1)</td>
</tr>
<tr>
<td>2)</td>
<td>2)</td>
</tr>
<tr>
<td>3)</td>
<td>3)</td>
</tr>
<tr>
<td>4)</td>
<td>4)</td>
</tr>
<tr>
<td>5)</td>
<td>5)</td>
</tr>
<tr>
<td>6)</td>
<td>6)</td>
</tr>
</tbody>
</table>
If you can show that two triangles are congruent, then you can show that all the corresponding angles and sides of the triangles are congruent.

**Problem**

Given: $AB \parallel DC$, $\angle B \cong \angle D$

Prove: $BC \cong DA$

In this case you know that $AB \parallel DC$, $AC$ forms a transversal and creates a pair of alternate interior angles, $\angle BAC$ and $\angle DCA$.

You have two pairs of congruent angles, $\angle BAC \cong \angle DCA$ and $\angle B \cong \angle D$. Because you know that the shared side is congruent to itself, you can use AAS to show that the triangles are congruent. Then use the fact that corresponding parts are congruent to show that $BC \cong DA$. Here is the proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $AB \parallel DC$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle BAC \cong \angle DCA$</td>
<td>2) Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>3) $\angle B \cong \angle D$</td>
<td>3) Given</td>
</tr>
<tr>
<td>4) $AC \cong CA$</td>
<td>4) Reflexive Property of Congruence</td>
</tr>
<tr>
<td>5) $\triangle ABC \cong \triangle DCA$</td>
<td>5) AAS</td>
</tr>
<tr>
<td>6) $BC \cong DA$</td>
<td>6) CPCTC</td>
</tr>
</tbody>
</table>

**Exercises**

1. Write a two-column proof.

Given: $MN \cong MP$, $NO \cong PO$

Prove: $\angle N \cong \angle P$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $?$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $MO \cong MO$</td>
<td>2) $?$</td>
</tr>
<tr>
<td>3) $?$</td>
<td>3) $?$</td>
</tr>
<tr>
<td>4) $\angle N \cong \angle P$</td>
<td>4) $?$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $?$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $MO \cong MO$</td>
<td>2) $?$</td>
</tr>
<tr>
<td>3) $?$</td>
<td>3) $?$</td>
</tr>
<tr>
<td>4) $\angle N \cong \angle P$</td>
<td>4) $?$</td>
</tr>
</tbody>
</table>
2. Write a two-column proof.
   Given: PT is a median and an altitude of ΔPRS.
   Prove: PT bisects ∠RPS.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) PT is a median of ΔPRS.</td>
<td>1) ?</td>
</tr>
<tr>
<td>2) ?</td>
<td>2) Definition of median</td>
</tr>
<tr>
<td>3) ?</td>
<td>3) Definition of midpoint</td>
</tr>
<tr>
<td>4) PT is an altitude of ΔPRS.</td>
<td>4) ?</td>
</tr>
<tr>
<td>5) PT ⊥ RS</td>
<td>5) ?</td>
</tr>
<tr>
<td>6) ∠PTS and ∠PTR are right angles.</td>
<td>6) ?</td>
</tr>
<tr>
<td>7) ?</td>
<td>7) All right angles are congruent.</td>
</tr>
<tr>
<td>8) ?</td>
<td>8) Reflexive Property of Congruence</td>
</tr>
<tr>
<td>9) ?</td>
<td>9) SAS</td>
</tr>
<tr>
<td>10) ∠TPS ≅ ∠TPR</td>
<td>10) ?</td>
</tr>
<tr>
<td>11) ?</td>
<td>11) ?</td>
</tr>
</tbody>
</table>

3. Write a two-column proof.
   Given: OQ ≅ OA; OQ bisects ∠KOA.
   Prove: KB ≅ AB

4. Write a two-column proof.
   Given: ON bisects ∠JOH, ∠J ≅ ∠JHS
   Prove: JN ≅ HN
4-4 Think About a Plan
Using Corresponding Parts of Congruent Triangles

**Constructions** The construction of $\angle B$ congruent to given $\angle A$ is shown. $AD \cong BF$ because they are the radii of the same circle. $DC \cong FE$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \cong \angle B$.

**Understanding the Problem**

1. What is the problem asking you to prove?

2. Segments $\overline{DC}$ and $\overline{FE}$ are not drawn on the construction. Draw them in. What figures are formed by drawing these segments?

3. What information do you need to be able to use corresponding parts of congruent triangles?

**Planning the Solution**

4. To use corresponding parts of congruent triangles, which two triangles do you need to show to be congruent?

5. What reason can you use to state that $\overline{AC} \cong \overline{BE}$?

**Getting an Answer**

6. Write a paragraph proof that uses corresponding parts of congruent triangles to prove that $\angle A \cong \angle B$. 


Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>base</strong></td>
<td>The <em>base</em> of an isosceles triangle is the side included between the pair of congruent angles.</td>
<td><img src="image" alt="Base" /></td>
</tr>
<tr>
<td><strong>base angles</strong></td>
<td>1.</td>
<td><img src="image" alt="Base Angles" /></td>
</tr>
<tr>
<td><strong>corollary</strong></td>
<td>2.</td>
<td>A corollary to the Isosceles Triangle Theorem is: If a triangle is equilateral, then the triangle is equiangular.</td>
</tr>
<tr>
<td><strong>equiangular triangle</strong></td>
<td>An <em>equiangular triangle</em> is a triangle with three congruent angles. The angles of an equiangular triangle all measure 60.</td>
<td><img src="image" alt="Equiangular Triangle" /></td>
</tr>
<tr>
<td><strong>Isosceles triangle</strong></td>
<td>4.</td>
<td><img src="image" alt="Isosceles Triangle" /></td>
</tr>
<tr>
<td><strong>legs</strong></td>
<td>The <em>legs</em> are the two congruent sides of an isosceles triangle.</td>
<td><img src="image" alt="Legs" /></td>
</tr>
<tr>
<td><strong>vertex angle</strong></td>
<td>6.</td>
<td><img src="image" alt="Vertex Angle" /></td>
</tr>
</tbody>
</table>
Two special types of triangles are isosceles triangles and equilateral triangles.

An isosceles triangle is a triangle with two congruent sides. The base angles of an isosceles triangle are also congruent. An altitude drawn from the shorter base splits an isosceles triangle into two congruent right triangles.

An equilateral triangle is a triangle that has three congruent sides and three congruent angles. Each angle measures 60°.

You can use the special properties of isosceles and equilateral triangles to find or prove different information about a given figure.

Look at the figure at the right.

You should be able to see that one of the triangles is equilateral and one is isosceles.

**Problem**

What is \( m\angle A \)?

\( \triangle ABC \) is isosceles because it has two base angles that are congruent. Because the sum of the measures of the angles of a triangle is 180, and \( m\angle B = 40 \), you can solve to find \( m\angle A \).

\[
\begin{align*}
  m\angle A + m\angle B + m\angle BEA &= 180 \\
  m\angle A + 40 + m\angle A &= 180 \\
  2m\angle A + 40 &= 180 \\
  2m\angle A &= 140 \\
  m\angle A &= 70
\end{align*}
\]

There are 180° in a triangle. Substitution Property

Combine like terms. Subtraction Property of Equality

Division Property of Equality

**Problem**

What is \( FC \)?

\( \triangle CFG \) is equilateral because it has three congruent angles.

\( CG = (2 + 2) = 4 \), and \( CG = FG = FC \).

So, \( FC = 4 \).
4-5 Reteaching (continued)

**Problem**
What is the value of $x$?
Because $x$ is the measure of an angle in an equilateral triangle, $x = 60$.

**Problem**
What is the value of $y$?

\[
\angle DCE + \angle DEC + \angle EDC = 180 \\
60 + 70 + y = 180 \\
y = 50
\]

**Exercises**
Complete each statement. Explain why it is true.

1. $\angle EAB \cong \ ?$
2. $\angle BCD \cong \ ? \cong \angle DBC$
3. $FG \cong \ ? \cong DF$

Determine the measure of the indicated angle.
4. $\angle ACB$
5. $\angle DCE$
6. $\angle BCD$

**Algebra** Find the value of $x$ and $y$.

7. 

8. 

9. **Reasoning** An exterior angle of an isosceles triangle has a measure 140.
   Find two possible sets of measures for the angles of the triangle.
4-5 Think About a Plan
Isosceles and Equilateral Triangles

Algebra The length of the base of an isosceles triangle is $x$. The length of a leg is $2x - 5$. The perimeter of the triangle is 20. Find $x$.

Know
1. What is the perimeter of a triangle?

2. What is an isosceles triangle?

Need
3. What are the sides of an isosceles triangle called?

4. How many of each type of side are there?

5. The lengths of the base and one leg are given. What is the third side of the triangle called?

Plan
6. Write an expression for the length of the third side.

7. Write an equation for the perimeter of this isosceles triangle.

8. Solve the equation for $x$. Show your work.
## Additional Vocabulary Support

The column on the left shows the steps used to prove that $\overline{AB} \cong \overline{ED}$. Use the column on the left to answer each question in the column on the right.

<table>
<thead>
<tr>
<th>Problem</th>
<th>[ \cong \text{ in Right Triangles} ]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Given: $C$ is the midpoint of $AE$ and $BD$</td>
<td>$\overline{AB} \parallel \overline{DE}$ and $\overline{BD}$</td>
<td>Prove: $\overline{AB} \cong \overline{ED}$</td>
</tr>
</tbody>
</table>

1. What is the definition of the midpoint of a line segment?

2. How do you know that $\overline{AB} \parallel \overline{DE}$ and $m\angle B = 90$?

3. What does the symbol $\cong$ between two line segments mean?

4. What does the word *interior* mean?

5. What is the measure of $\angle D$?

6. What information is necessary to apply the HL Postulate?

7. What are the corresponding angles and sides for $\triangle ABC$ and $\triangle EDC$?
Two right triangles are congruent if they have congruent hypotenuses and if they have one pair of congruent legs. This is the **Hypotenuse-Leg (HL) Theorem**.

\[ \triangle ABC \cong \triangle PQR \] because they are both right triangles, their hypotenuses are congruent \((AC \cong PR)\), and one pair of legs is congruent \((BC \cong QR)\).

**Problem**

How can you prove that two right triangles that have one pair of congruent legs and congruent hypotenuses are congruent (The Hypotenuse-Leg Theorem)?

Both of the triangles are right triangles.

\[ \angle B \text{ and } \angle E \] are right angles.

\[ AB \cong DE \] and \[ AC \cong DF \].

How can you prove that \( \triangle ABC \cong \triangle DEF \)?

Look at \( \triangle DEF \). Draw a ray starting at \( F \) that passes through \( E \). Mark a point \( X \) so that \( EX = BC \). Then draw \( DX \) to create \( \triangle DEX \).

See that \( EX \cong BC \). (You drew this.) \( \angle DEX \cong \angle ABC \). (All right angles are congruent.) \( DE \cong AB \). (This was given.) So, by SAS, \( \triangle ABC \cong \triangle DEX \).

\[ DX \cong AC \] (by CPCTC) and \( AC \cong DF \). (This was given.). So, by the Transitive Property of Congruence, \( DX \cong DF \). Then, \( \angle DEX \cong \angle DEF \). (All right angles are congruent.) By the Isosceles Theorem, \( \angle X \cong \angle F \). So, by AAS, \( \triangle DEX \cong \triangle DEF \).

Therefore, by the Transitive Property of Congruence, \( \triangle ABC \cong \triangle DEF \).

**Problem**

Are the given triangles congruent by the Hypotenuse-Leg Theorem? If so, write the triangle congruence statement.

\[ \angle F \text{ and } \angle H \] are both right angles, so the triangles are both right.

\[ GI \cong IG \] by the Reflexive Property and \( FI \cong HG \) is given.

So, \( \triangle FIG \cong \triangle HGI \).
Exercises

Determine if the given triangles are congruent by the Hypotenuse-Leg Theorem. If so, write the triangle congruence statement.

1. \(\triangle TUV \cong \triangle LMN\)

2. \(\triangle RST \cong \triangle XYZ\)

3. \(\triangle LMQ \cong \triangle SNR\)

4. \(\triangle DOH \cong \triangle PQR\)

Measure the hypotenuse and length of the legs of the given triangles with a ruler to determine if the triangles are congruent. If so, write the triangle congruence statement.

5. \(\triangle ABC\)

6. \(\triangle DEF\)

7. Explain why \(\triangle LMN \cong \triangle OMQ\). Use the Hypotenuse-Leg Theorem.

8. Visualize \(\triangle ABC\) and \(\triangle DEF\), where \(AB = EF\) and \(CA = FD\). What else must be true about these two triangles to prove that the triangles are congruent using the Hypotenuse-Leg Theorem? Write a congruence statement.
Think About a Plan

Congruence in Right Triangles

Algebra For what values of $x$ and $y$ are the triangles congruent by HL?

Know

1. For two triangles to be congruent by the Hypotenuse-Leg Theorem, there must be a ____________, and the lengths of ____________ and ____________ must be equal.

2. The length of the hypotenuse of the triangle on the left is ____________ and the hypotenuse of the triangle on the right is ____________.

3. The length of the leg of the triangle on the left is ____________ and the length of the leg of the triangle on the right is ____________.

Need

4. To solve the problem you need to find ____________.

Plan

5. What system of equations can you use to find the values of $x$ and $y$?

6. What method(s) can you use to solve the system of equations?

7. What is the value of $y$? What is the value of $x$?
A student wanted to prove $EB \cong DB$ given $\angle AED \cong \angle CDE$ and $AE \cong CD$. She wrote the statements and reasons on note cards, but they got mixed up.

Use the note cards to write the steps in order.

1. First, ____________________________
2. Second, ____________________________
3. Third, ____________________________
4. Next, ____________________________
5. Then, ____________________________
6. Then, ____________________________
7. Finally, ____________________________
Sometimes you can prove one pair of triangles congruent and then use corresponding parts of those triangles to prove another pair congruent. Often the triangles overlap.

**Problem**

Given: \( \overline{AB} \cong \overline{CB} \),  
\( \overline{AE} \cong \overline{CD} \),  
\( \angle AED \cong \angle CDE \)

Prove: \( \triangle ABE \cong \triangle CBD \)

Think about a plan for the proof. Examine the triangles you are trying to prove congruent. Two pairs of sides are congruent. If the included angles, \( \angle A \) and \( \angle C \), were congruent, then the triangles would be congruent by SAS.

If the overlapping triangles \( \triangle AED \) and \( \triangle CDE \) were congruent, then the angles would be congruent by corresponding parts. When triangles overlap, sometimes it is easier to visualize if you redraw the triangles separately.

Now use the plan to write a proof.

Given: \( \overline{AB} \cong \overline{CB}, \overline{AE} \cong \overline{CD}, \angle AED \cong \angle CDE \)

Prove: \( \triangle ABE \cong \triangle CBD \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \overline{AE} \cong \overline{CD}, \angle AED \cong \angle CDE )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \overline{ED} \cong \overline{ED} )</td>
<td>2) Reflexive Property of ( \cong )</td>
</tr>
<tr>
<td>3) ( \triangle AED \cong \triangle CDE )</td>
<td>3) SAS</td>
</tr>
<tr>
<td>4) ( \angle A \cong \angle C )</td>
<td>4) CPCTC</td>
</tr>
<tr>
<td>5) ( \overline{AB} \cong \overline{CB} )</td>
<td>5) Given</td>
</tr>
<tr>
<td>6) ( \triangle ABE \cong \triangle CBD )</td>
<td>6) SAS</td>
</tr>
</tbody>
</table>
Reteaching (continued)

Separate and redraw the overlapping triangles. Identify the vertices.

1. \(\triangle ALJ\) and \(\triangle HJL\)

2. \(\triangle MRP\) and \(\triangle NQS\)

3. \(\triangle FED\) and \(\triangle CDE\)

Fill in the blanks for the two-column proof.

4. Given: \(\angle AEG = \angle AFD, \overline{AE} = \overline{AF}, \overline{GE} = \overline{FD}\)
Prove: \(\triangle AFG \cong \triangle AED\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) (\angle AEG = \angle AFD, \overline{AE} = \overline{AF}, \overline{GE} = \overline{FD})</td>
<td>1) ?</td>
</tr>
<tr>
<td>2) ?</td>
<td>2) SAS</td>
</tr>
<tr>
<td>3) (\overline{AG} = \overline{AD}, \angle G = \angle D)</td>
<td>3) ?</td>
</tr>
<tr>
<td>4) ?</td>
<td>4) Given</td>
</tr>
<tr>
<td>5) (\overline{GE} = \overline{FD})</td>
<td>5) ?</td>
</tr>
<tr>
<td>6) (\overline{GF} + \overline{FE} = \overline{GE}, \overline{FE} + \overline{ED} = \overline{FD})</td>
<td>6) ?</td>
</tr>
<tr>
<td>7) (\overline{GF} + \overline{FE} = \overline{FE} + \overline{ED})</td>
<td>7) ?</td>
</tr>
<tr>
<td>8) ?</td>
<td>8) Subtr. Prop. of Equality</td>
</tr>
<tr>
<td>9) ?</td>
<td>9) ?</td>
</tr>
</tbody>
</table>

Use the plan to write a two-column proof.

5. Given: \(\angle PSR\) and \(\angle PQR\) are right angles, \(\angle QPR = \angle SRP\).
Prove: \(\triangle STR \cong \triangle TQP\)

Plan for Proof:

Prove \(\triangle QPR \cong \triangle SRP\) by AAS. Then use CPCTC and vertical angles to prove \(\triangle STR \cong \triangle TQP\) by AAS.
4-7 Think About a Plan

Congruence in Overlapping Triangles

Given: $ QT \perp PR$, $ QT$ bisects $ PR$, $ QT$ bisects $ \angle VQS$

Prove: $ VQ \cong SQ$

Know

1. What information are you given? What else can you determine from the given information and the diagram?

2. To solve the problem, what will you need to prove?

Need

3. For which two triangles are $ VQ$ and $ SQ$ corresponding parts?

4. You need to use corresponding parts to prove the triangles from Exercise 3 congruent. Which two triangles should you prove congruent first, using the given information? Which theorem or postulate should you use?

5. Which corresponding parts should you then use to prove that the triangles in Exercise 3 are congruent?

Plan

6. Use the space below to write the proof:
Answer Keys
### Concept List

<table>
<thead>
<tr>
<th>algebraic equation</th>
<th>angle measure</th>
<th>congruency statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>congruent angles</td>
<td>congruent polygons</td>
<td>congruent segments</td>
</tr>
<tr>
<td>congruent triangles</td>
<td>proof</td>
<td>segment measure</td>
</tr>
</tbody>
</table>

Choose the concept from the list above that best represents the item in each box.

1. $\overline{GH} \cong \overline{ST}$
   - congruency statement

2. $m \angle A = 45$
   - angle measure

3. [Diagram of congruent polygons]
   - congruent polygons

4. $YZ = MN$
   - congruent segments

5. $\triangle ABC \cong \triangle XYZ$
   - congruent triangles

6. Given: $\overline{BD}$ is the angle bisector of $\angle ABC$, and $\overline{BD}$ is the perpendicular bisector of $AC$.  
   Prove: $\triangle ADB \cong \triangle CDB$
   - proof

7. $m \angle H = 5x$  
   $m \angle W = x + 28$
   Solve $5x = x + 28$ to find the measures of $\angle H$ and $\angle W$.
   - algebraic equation

8. $BC = 3 \text{ cm}$
   - segment measure

9. $\angle ADB$ and $\angle SDT$ are vertical angles. So, $\angle ADB \cong \angle SDT$.
   - congruent angles
4-1  
Reteaching
Congruent Figures

Given $ABCD \cong QRST$, find corresponding parts using the names. Order matters.

For example, $\triangle ABCD$  
$\triangle QRST$
This shows that $\angle A$ corresponds to $\angle Q$.
Therefore, $\angle A \cong \angle Q$.

For example, $\overline{ABCD}$  
$\overline{QRST}$
This shows that $\overline{BC}$ corresponds to $\overline{RS}$.
Therefore, $\overline{BC} \cong \overline{RS}$.

Exercises

Find corresponding parts using the order of the letters in the names.

1. Identify the remaining three pairs of corresponding angles and sides between $ABCD$ and $QRST$ using the circle technique shown above.

$\angle B \cong \angle R$, $\angle C \cong \angle S$, $\angle D \cong \angle T$, $\overline{AB} \cong \overline{QR}$, $\overline{CD} \cong \overline{ST}$, and $\overline{DA} \cong \overline{TQ}$

Angles: $ABCD$  $ABCD$  $ABCD$
Sides: $ABCD$  $ABCD$  $ABCD$

$QRST$  $QRST$  $QRST$

2. Which pair of corresponding sides is hardest to identify using this technique?

Answers may vary. Sample: $\overline{AD}$ and $\overline{QT}$

Find corresponding parts by redrawing figures.

3. The two congruent figures below at the left have been redrawn at the right. Why are the corresponding parts easier to identify in the drawing at the right?

4. Redraw the congruent polygons at the right in the same orientation. Identify all pairs of corresponding sides and angles. Check students’ work. $\angle A$ and $\angle P$, $\overline{AB}$ and $\overline{PQ}$, $\angle C$ and $\angle Q$, $\angle D$ and $\angle S$, $\angle E$ and $\angle T$, $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$, $\overline{CD}$ and $\overline{RS}$, $\overline{DE}$ and $\overline{ST}$, and $\overline{EA}$ and $\overline{TP}$ all correspond.

5. $MNOP \cong QRST$. Identify all pairs of congruent sides and angles.

$\angle M \cong \angle Q$, $\angle N \cong \angle R$, $\angle O \cong \angle S$, $\angle P \cong \angle T$, $\overline{MN} \cong \overline{QR}$, $\overline{NO} \cong \overline{RS}$, $\overline{OP} \cong \overline{ST}$, and $\overline{PM} \cong \overline{TQ}$
4-1

Reteaching (continued)

Congruent Figures

Problem

Given \( \triangle ABC \cong \triangle DEF \), \( m \angle A = 30 \), and \( m \angle E = 65 \), what is \( m \angle C \)?

How might you solve this problem? Sketch both triangles, and put all the information on both diagrams.

\( m \angle A = 30 \); therefore, \( m \angle D = 30 \). How do you know?
Because \( \angle A \) and \( \angle D \) are corresponding parts of congruent triangles.

Exercises

Work through the exercises below to solve the problem above.

6. What angle in \( \triangle ABC \) has the same measure as \( \angle E \)? What is the measure of that angle? Add the information to your sketch of \( \triangle ABC \).
   \( \angle B; 65 \)

7. You know the measures of two angles in \( \triangle ABC \). How can you find the measure of the third angle?
   Answers may vary. Sample: Use Triangle Angle-Sum Thm. Set sum of all three angles equal to 180.

8. What is \( m \angle C \)? How did you find your answer?
   \( 85 \); answers may vary. Sample: \( m \angle C = 180 - (60 + 35) \)

Before writing a proof, add the information implied by each given statement to your sketch. Then use your sketch to help you with Exercises 9–12.

Add the information implied by each given statement.

9. Given: \( \angle A \) and \( \angle C \) are right angles.
   \( m \angle A = m \angle C = 90 \), \( \overline{DA} \parallel \overline{AB} \) and \( \overline{DC} \parallel \overline{BC} \)

10. Given: \( \overline{AB} \cong \overline{CD} \) and \( \overline{AD} \cong \overline{CB} \).
    \( \triangle ABCD \) is a parallelogram because it has opposite sides that are congruent.

11. Given: \( \angle ADB \cong \angle CBD \).
    \( \overline{AB} \parallel \overline{BC} \)

12. Can you conclude that \( \angle ABD \cong \angle CDB \) using the given information above?
    If so, how?
    Yes; use the Third Angles Thm.

13. How can you conclude that the third side of both triangles is congruent?
    The third side is shared by both triangles and is congruent by the Refl. Prop. of Congruence.
Think About a Plan

Congruent Figures

Algebra Find the values of the variables.

Know

1. What do you know about the measure of each of the non-right angles?
   The measure of each of the non-right angles are complementary.

2. What do you know about the length of each of the legs?
   All of the legs are equal in length.

3. What types of triangles are shown in the figure?
   isosceles right triangles

Need

4. What information do you need to know to find the value of \(x\)?
   You need to know that the measure of each of the non-right angles is 45.

5. What information do you need to know to find the value of \(t\)?
   You need to know that the length of each of the legs is 4 in.

Plan

6. How can you find the value of \(x\)? What is its value?
   Answers may vary. Sample: Set \(3x\) equal to 45. So, \(3x = 45\); \(x = 15\).

7. How do you find the value of \(t\)? What is its value?
   Answers may vary. Sample: Set \(2t\) equal to 4. So, \(2t = 4\); \(t = 2\).
4-2 Additional Vocabulary Support
Triangle Congruence by SSS and SAS

Problem
Use the figure at the right. How can you prove that \( \triangle ABC \cong \triangle XYZ \)? Justify each step.

Given: The figure at the right

Prove: \( \triangle ABC \cong \triangle XYZ \)
1) \( AB \equiv XY \)  
   1) Given
2) \( BC \equiv YZ \)  
   2) Given
3) \( \angle A \equiv \angle X \)  
   3) Given
4) \( \angle C \equiv \angle Z \)  
   4) Given
5) \( \angle B \equiv \angle Y \)  
   5) Third Angles Theorem
6) \( \triangle ABC \cong \triangle XYZ \)  
   6) Side-Angle-Side (SAS) Postulate

Exercises

1. Use the figure at the right. How can you prove that \( \triangle GMH \cong \triangle TMS \)? Justify each step.

Given: \( M \) is the midpoint of \( HS \) and \( GT \).

Prove: \( \triangle GMH \cong \triangle TMS \)
1) \( M \) is the midpoint of \( HS \) and \( GT \).  
   1) Given
2) \( GM \equiv TM \)  
   2) Definition of the midpoint
3) \( HM \equiv SM \)  
   3) Definition of the midpoint
4) \( \angle GMH \equiv \angle TMS \)  
   4) Vertical angles are congruent.
5) \( \triangle GMH \equiv \triangle TMS \)  
   5) Side-Angle-Side (SAS) Postulate

2. Use the figure at the right. How can you prove that \( \triangle GHI \cong \triangle JHI \)? Justify each step.

Given: \( H \) is the midpoint of \( GI \).

Prove: \( \triangle GHI \cong \triangle JHI \)
1) \( H \) is the midpoint of \( GI \).  
   1) Given
2) \( GH \equiv JH \)  
   2) Reflexive Property of \( \cong \)
3) \( HI \equiv HI \)  
   3) Third Side Postulate
4) \( GI \equiv JI \)  
   4) Side-Side Side (SSS) Postulate
5) \( \triangle GHI \cong \triangle JHI \)  
   5)
4-2  Keteaching
Triangle Congruence by SSS and SAS

You can prove that triangles are congruent using the two postulates below.

**Postulate 4-1: Side-Side-Side (SSS) Postulate**
If all three sides of a triangle are congruent to all three sides of another triangle, then those two triangles are congruent.

If $\overline{JK} \cong \overline{XY}$, $\overline{KL} \cong \overline{YZ}$, and $\overline{JL} \cong \overline{XZ}$, then $\triangle JKL \cong \triangle XYZ$.

In a triangle, the angle formed by any two sides is called the included angle for those sides.

**Postulate 4-2: Side-Angle-Side (SAS) Postulate**
If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then those two triangles are congruent.

If $\overline{PQ} \cong \overline{DE}$, $\overline{PR} \cong \overline{DF}$, and $\angle P \cong \angle D$, then $\triangle PQR \cong \triangle DEF$.

$\angle P$ is included by $\overline{QP}$ and $\overline{PR}$. $\angle D$ is included by $\overline{ED}$ and $\overline{DF}$.

**Exercises**

1. What other information do you need to prove $\triangle TRF \cong \triangle DFR$ by SAS? Explain. $DF = TR$; by the Reflexive Property of Congruence, $RF = FR$. It is given that $\angle TRF \cong \angle DFR$. These are the included angles for the corresponding congruent sides.

2. What other information do you need to prove $\triangle ABC \cong \triangle DEF$ by SAS? Explain. $\angle B = \angle E$; These are the included angles between the corresponding congruent sides.

3. **Developing Proof** Copy and complete the flow proof.
   Given: $\overline{DA} \cong \overline{MA}$, $\overline{AJ} \cong \overline{AZ}$
   Prove: $\triangle JDA \cong \triangle ZMA$
Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information.* Explain your answer.

4. Not enough information; two pairs of corresponding sides are congruent, but the congruent angles are not the included angles.

5. Not enough information; you need to know if $GC = DY$.

6. Not enough information; only two pairs of corresponding sides are congruent. You need to know if $AB = XY$ or $LZ = LC$.

7. Given: $\overline{PO} \equiv \overline{SO}$, $O$ is the midpoint of $NT$.

Prove: $\triangle NOP \equiv \triangle TOS$

Statements: 1) $\overline{PO} \equiv \overline{SO}$; 2) $O$ is the midpoint of $NT$; 3) $NO \equiv TO$;
4) $\angle NOP \equiv \angle TOS$; 5) $\triangle NOP \equiv \triangle TOS$;
Reasons: 1) Given; 2) Given; 3) Def. of midpoint; 4) Vert. $\angle$ are =; 5) SAS

8. Given: $\overline{HI} \equiv \overline{HG}$, $FH \perp GI$

Prove: $\triangle FHI \equiv \triangle FHG$

Statements: 1) $\overline{FH} \equiv \overline{FH}$; 2) $\overline{HI} \equiv \overline{HG}$, $FH \perp GI$; 3) $\angle FHG$ and $\angle FHI$ are rt. $\angle$s;
4) $\angle FHG \equiv \angle FHI$; 5) $\triangle FHI \equiv \triangle FHG$;
Reasons: 1) Refl. Prop.; 2) Given; 3) Def. of perpendicular; 4) All rt. $\angle$ are $\equiv$; 5) SAS

9. A carpenter is building a support for a bird feeder. He wants the triangles on either side of the vertical post to be congruent. He measures and finds that $\overline{AB} \equiv \overline{DE}$ and that $\overline{AC} \equiv \overline{DF}$.
What would he need to measure to prove that the triangles are congruent using SAS? What would he need to measure to prove that they are congruent using SSS?
For SAS, he would need to determine if $\angle BAC \equiv \angle EDF$; for SSS, he would need to determine if $\overline{BC} \equiv \overline{EF}$.

10. An artist is drawing two triangles. She draws each so that two sides are 4 in. and 5 in. long and an angle is $55^\circ$. Are her triangles congruent? Explain.
Answers may vary. Sample: Maybe; if both the $55^\circ$ angles are between the 4-in. and 5-in. sides, then the triangles are congruent by SAS.
Think About a Plan
Triangle Congruence by SSS and SAS

Use the Distance Formula to determine whether \( \triangle ABC \) and \( \triangle DEF \) are congruent. Justify your answer.

\[
A(1, 4), \ B(5, 5), \ C(2, 2)
\]
\[
D(5, 1), \ E(1, 0), \ F(4, 3)
\]

Understanding the Problem

1. You need to determine if \( \triangle ABC \equiv \triangle DEF \). What are the three ways you know to prove triangles congruent?
   - If all corresponding parts are congruent, if all three sides are congruent, or, if two sides and the included angle are congruent, then the triangles are congruent.

2. What information is given in the problem?
   - the coordinates for each vertex of each triangle

Planning the Solution

3. If you use the SSS Postulate to determine whether the triangles are congruent, what information do you need to find?
   - the lengths of the three sides of each triangle

4. How can you find distances on a coordinate plane without measuring?
   - Use the Distance Formula.

5. In an ordered pair, which number is the x-coordinate? Which is the y-coordinate?
   - The x-coordinate is the first number and the y-coordinate is the second number.

Getting an Answer

6. Find the length of each segment using the Distance Formula,
   \[
   D = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}
   \]
   - Your answers may be in simplest radical form.

\[
\begin{align*}
\overline{AB} & = \sqrt{17} \\
\overline{BC} & = 3\sqrt{2} \\
\overline{CA} & = \sqrt{5} \\
\overline{DE} & = \sqrt{17} \\
\overline{EF} & = 3\sqrt{2} \\
\overline{FD} & = \sqrt{5}
\end{align*}
\]

7. Using the SSS Postulate, are the triangles congruent? Explain.
   - Yes; the triangles are congruent, because three pairs of sides are congruent.
### Additional Vocabulary Support

**Triangle Congruence by ASA and AAS**

#### Problem

Given: The figure at the right
Prove: \( \triangle ABQ \cong \triangle XYQ \)

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
</table>
| First, list the information that is given directly in the diagram. | \( \overline{BQ} \cong \overline{YQ} \)  
\( \angle A \) and \( \angle X \) are right \( \triangle \). | Given |
| Second, use the fact that all right angles are congruent to each other. | \( \angle A \cong \angle X \) | All right angles are congruent. |
| Next, use the fact that vertical angles are congruent. | \( \angle AQB \cong \angle XQY \) | Vertical Angles Theorem |
| Finally, determine which theorem can be used to prove the triangles congruent using the information listed above. | \( \triangle ABQ \cong \triangle XYQ \) | Angle-Angle-Side (AAS) Theorem |

#### Exercise

Given: The figure at the right
Prove: \( \triangle HJL \cong \triangle KIJ \)

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
</table>
| First, list the information that is given directly in the diagram. | \( \overline{HJ} \parallel \overline{KL} \)  
\( \overline{HL} \parallel \overline{KJ} \) | Given |
| Second, use the fact that alternate interior angles are congruent when parallel lines are cut by a transversal. | \( \angle HJL \cong \angle KIJ \)  
\( \angle HLJ \cong \angle KJL \) | Alternate Interior Angles Theorem |
| Next, recall that any line segment is congruent to itself. | \( \overline{HL} \cong \overline{IL} \) | Reflexive Property of Congruence |
| Finally, determine which theorem can be used to prove the triangles congruent using the information listed above. | \( \triangle HJL \cong \triangle KIJ \) | Angle-Side-Angle (ASA) Postulate |
4-3  
Reteaching  
Triangle Congruence by ASA and AAS

**Problem**

Can the ASA Postulate or the AAS Theorem be applied directly to prove the triangles congruent?

![Diagram](image)

**Exercises**

Indicate congruences.

1. Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.

2. Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.

3. Draw and mark two triangles that are congruent by either the ASA Postulate or the AAS Theorem. Check students' work.

What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?

- **4.** ASA Postulate  
  \( \angle ABD = \angle CBD \)

- **5.** AAS Theorem  
  \( \angle JMK = \angle LKM, \)  
  \( \angle JMK = \angle LMK, \)  
  \( \angle JMK = \angle LMK, \)  
  or  
  \( \angle JKM = \angle LKM \)

- **6.** ASA Postulate  
  \( \angle XYZ = \angle ZUV \)

- **7.** AAS Theorem  
  \( \angle Y = \angle Z \)

- **8.** AAS Theorem  
  \( \angle L = \angle A \)

- **9.** ASA Postulate  
  \( \angle CYL = \angle ALC \)
10. Provide the reason for each step in the two-column proof.
Given: $TX \parallel VW$, $TU \equiv VU$, $\angle XTU \equiv \angle WVU$, $\angle UWV$ is a right angle.
Prove: $\triangle TUX \equiv \triangle VUW$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $\angle UWV$ is a right angle.</td>
<td>1) ? Given</td>
</tr>
<tr>
<td>2) $VW \perp UW$</td>
<td>2) ? Definition of perpendicular lines</td>
</tr>
<tr>
<td>3) $TX \parallel VW$</td>
<td>3) ? Given</td>
</tr>
<tr>
<td>4) $TX \perp UW$</td>
<td>4) ? Perpendicular Transversal Theorem</td>
</tr>
<tr>
<td>5) $\angle UXT$ is a right angle.</td>
<td>5) ? Definition of perpendicular lines</td>
</tr>
<tr>
<td>6) $\angle UWV \equiv \angle UXT$</td>
<td>6) ? All right angles are congruent.</td>
</tr>
<tr>
<td>7) $TU \equiv VU$</td>
<td>7) ? Given</td>
</tr>
<tr>
<td>8) $\angle XTU \equiv \angle WVU$</td>
<td>8) ? Given</td>
</tr>
<tr>
<td>9) $\triangle TUX \equiv \triangle VUW$</td>
<td>9) ? AAS Theorem</td>
</tr>
</tbody>
</table>

11. Write a paragraph proof.
Given: $WX \parallel ZY$; $WZ \parallel XY$
Prove: $\triangle XWY \equiv \triangle YZW$

It is given that $WX \parallel ZY$ and $WZ \parallel XY$, so $\angle XWY \equiv \angle ZYW$ and $\angle XYW \equiv \angle ZYW$, by the Alternate Interior $\triangle$ Thm. $WY \equiv YW$ by the Reflexive Property of $\sim$. So, by ASA Post. $\triangle XWY \equiv \triangle YZW$.

12. Developing Proof
Complete the proof by filling in the blanks.
Given: $\angle A \equiv \angle C$, $\angle 1 \equiv \angle 2$
Prove: $\triangle ABD \equiv \triangle CDB$
Proof: $\angle A \equiv \angle C$ and $\angle 1 \equiv \angle 2$ are given. $\overline{DB} \equiv \overline{BD}$ by ?

So, $\triangle ABD \equiv \triangle CDB$ by ? AAS

13. Write a paragraph proof.
Given: $\angle 1 \equiv \angle 6$, $\angle 3 \equiv \angle 4$, $\overline{LP} \equiv \overline{OP}$
Prove: $\triangle LMP \equiv \triangle ONP$

$\angle 3 \equiv \angle 4$ is given. Therefore, $m\angle 3 = m\angle 4$, by def. of $\equiv \angle$s. Because $\angle 2$ and $\angle 3$ are linear pairs, and $\angle 4$ and $\angle 5$ are linear pairs, the pairs of angles are suppl. Therefore, $\angle 2 \equiv \angle 5$ by the Congruent Suppl. Thm. $\angle 1 \equiv \angle 6$ and $\overline{LP} \equiv \overline{OP}$ are given, so $\triangle LMP \equiv \triangle ONP$, by the AAS Thm.
Think About a Plan
Triangle Congruence by ASA and AAS

Given: \(AB \parallel CD, AD \parallel CB\)

Prove: \(\triangle ABC \cong \triangle CDA\)

1. What do you need to find to solve the problem?
   Sample: at least three corresponding pairs of sides or angles that I can prove
to be congruent

2. What are the corresponding parts of the two triangles?
   \(\angle CAB\) and \(\angle ACD\); \(\angle D\) and \(\angle B\); \(\angle DAC\) and \(\angle BCA\); \(\overline{AC}\) and \(\overline{CA}\); \(\overline{AB}\) and \(\overline{CD}\);
and \(\overline{BC}\) and \(\overline{DA}\)

3. What word would you use to describe \(\overline{AC}\)? transversal

4. What can you show about angles in the triangles that can indicate congruency?
   I can find congruent angles using alternate interior angles of the transversal \(\overline{AC}\).

5. What do you know about a side or sides of the triangles that can be used to show congruency?
The transversal is part of both triangles, so it is congruent to itself by the
Reflexive Property of Congruence.

6. Write a proof in paragraph form.
   Answers may vary. Sample: \(AB \parallel DC\) and \(AD \parallel BC\) are given. Therefore \(\overline{AC}\) is a
transversal. \(\angle CAB \cong \angle ACD\) and \(\angle DAC \cong \angle BCA\) by the Alternate Interior Angles
Theorem. The transversal is part of both triangles, so \(\overline{AC} \cong \overline{CA}\) by the Reflexive
Property of Congruence. \(\triangle ABC \cong \triangle CDA\) by the ASA Postulate.
4-4  Additional Vocabulary Support

Using Corresponding Parts of Congruent Triangles

There are two sets of note cards below that show how to prove $BD$ is the perpendicular bisector of $AE$. The set on the left has the statements and the set on the right has the reasons. Write the statements and the reasons in the correct order.

**Statements**

1. $\angle BAC \cong \angle DEC$
2. $AC = EC$
3. $\triangle ACB \cong \triangle ECD$
4. $BD$ is the perpendicular bisector of $AE$.
5. $BC = DC; \angle ACB$ and $\angle ECD$ are right angles; $AB \parallel DE$
6. $\angle ACB \cong \angle ECD$

**Reasons**

1. Definition of the perpendicular bisector
2. Angle-Angle-Side (AAS) Theorem
3. When parallel lines are cut by a transversal, alternate interior angles are congruent.
4. Corresponding parts of congruent triangles are congruent.
5. Given
6. All right angles are congruent.

**Statements**

1) $BC = DC; \angle ACB$ and $\angle ECD$ are right angles; $AB \parallel DE$
2) $\angle ACB \cong \angle ECD$
3) $\angle BAC \cong \angle DEC$
4) $\triangle ACB \cong \triangle ECD$
5) $AC = EC$
6) $BD$ is the perpendicular bisector of $AE$.

**Reasons**

1) Given
2) All right angles are congruent.
3) When parallel lines are cut by a transversal, alternate interior angles are congruent.
4) Angle-Angle-Side (AAS) Theorem
5) Corresponding parts of congruent triangles are congruent.
6) Definition of the perpendicular bisector
4-4 Reteaching
Using Corresponding Parts of Congruent Triangles

If you can show that two triangles are congruent, then you can show that all the corresponding angles and sides of the triangles are congruent.

**Problem**

Given: \( AB \parallel DC \), \( \angle B \equiv \angle D \)

Prove: \( BC \equiv DA \)

In this case you know that \( AB \parallel DC \). \( AC \) forms a transversal and creates a pair of alternate interior angles, \( \angle BAC \) and \( \angle DCA \).

You have two pairs of congruent angles, \( \angle BAC \equiv \angle DCA \) and \( \angle B \equiv \angle D \). Because you know that the shared side is congruent to itself, you can use AAS to show that the triangles are congruent. Then use the fact that corresponding parts are congruent to show that \( BC \equiv DA \). Here is the proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( AB \parallel DC )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle BAC \equiv \angle DCA )</td>
<td>2) Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>3) ( \angle B \equiv \angle D )</td>
<td>3) Given</td>
</tr>
<tr>
<td>4) ( AC \equiv CA )</td>
<td>4) Reflexive Property of Congruence</td>
</tr>
<tr>
<td>5) ( \triangle ABC \equiv \triangle CDA )</td>
<td>5) AAS</td>
</tr>
<tr>
<td>6) ( BC \equiv DA )</td>
<td>6) CPCTC</td>
</tr>
</tbody>
</table>

**Exercises**

1. Write a two-column proof.

Given: \( MN \equiv MP \), \( NO \equiv PO \)

Prove: \( \angle N \equiv \angle P \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ? ( MN \equiv MP ), ( NO \equiv PO )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( MO \equiv MO )</td>
<td>2) ? Reflexive Property of ( \equiv )</td>
</tr>
<tr>
<td>3) ? ( \triangle MNO \equiv \triangle MPO )</td>
<td>3) ? SSS</td>
</tr>
<tr>
<td>4) ( \angle N \equiv \angle P )</td>
<td>4) ? CPCTC</td>
</tr>
</tbody>
</table>
2. Write a two-column proof.

   Given: $PT$ is a median and an altitude of $\triangle PRS$.
   Prove: $PT$ bisects $\angle RPS$.  

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $PT$ is a median of $\triangle PRS.$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $T$ is the midpoint of $RS$.</td>
<td>2) Definition of median</td>
</tr>
<tr>
<td>3) $RT = ST$</td>
<td>3) Definition of midpoint</td>
</tr>
<tr>
<td>4) $PT$ is an altitude of $\triangle PRS$.</td>
<td>4) Given</td>
</tr>
<tr>
<td>5) $PT \perp RS$</td>
<td>5) Definition of altitude</td>
</tr>
<tr>
<td>6) $\angle PTS$ and $\angle PTR$ are right angles.</td>
<td>6) Definition of perpendicular</td>
</tr>
<tr>
<td>7) $\angle PTS = \angle PTR$</td>
<td>7) All right angles are congruent.</td>
</tr>
<tr>
<td>8) $PT = PT$</td>
<td>8) Reflexive Property of Congruence</td>
</tr>
<tr>
<td>9) $\triangle PTS = \triangle PTR$</td>
<td>9) SAS</td>
</tr>
<tr>
<td>10) $\angle TPS \equiv \angle TPR$</td>
<td>10) CPCTC</td>
</tr>
<tr>
<td>11) $PT$ bisects $\angle RPS$.</td>
<td>11) Definition of angle bisector</td>
</tr>
</tbody>
</table>

3. Write a two-column proof.

   Given: $QK \equiv QA; QB$ bisects $\angle KQA$.
   Prove: $KB \equiv AB$  

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $QK \equiv QA; QB$ bisects $\angle KQA$.</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle KQB = \angle AQB$</td>
<td>2) Def. of $\angle$ bis.</td>
</tr>
<tr>
<td>3) $BQ = BQ$</td>
<td>3) Refl. Prop. of Congruence</td>
</tr>
<tr>
<td>4) $\triangle KBQ \equiv \triangle ABQ$</td>
<td>4) SAS</td>
</tr>
<tr>
<td>5) $KB \equiv AB$</td>
<td>5) CPCTC</td>
</tr>
</tbody>
</table>

4. Write a two-column proof.

   Given: $ON$ bisects $\angle JOH, \angle J \equiv \angle H$
   Prove: $JN \equiv HN$  

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $ON$ bisects $\angle JOH, \angle J \equiv \angle H$</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle JON = \angle HON$</td>
<td>2) Def. of $\angle$ bis.</td>
</tr>
<tr>
<td>3) $\angle ON = \angle ON$</td>
<td>3) Refl. Prop. of Congruence</td>
</tr>
<tr>
<td>4) $\triangle JON = \triangle HON$</td>
<td>4) AAS</td>
</tr>
<tr>
<td>5) $JN = HN$</td>
<td>5) CPCTC</td>
</tr>
</tbody>
</table>
Think About a Plan
Using Corresponding Parts of Congruent Triangles

Constructions The construction of $\angle B$ congruent to given $\angle A$ is shown. $AD \equiv BF$ because they are the radii of the same circle. $DC \equiv FE$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \equiv \angle B$.

Understanding the Problem

1. What is the problem asking you to prove?
   $\angle A \equiv \angle B$

2. Segments $\overline{DC}$ and $\overline{FE}$ are not drawn on the construction. Draw them in. What figures are formed by drawing these segments?
   two triangles

3. What information do you need to be able to use corresponding parts of congruent triangles?
   $\overline{AC}$ needs to be shown as congruent to $\overline{BE}$.

Planning the Solution

4. To use corresponding parts of congruent triangles, which two triangles do you need to show to be congruent?
   $\triangle ACD$ and $\triangle BFE$

5. What reason can you use to state that $\overline{AC} \equiv \overline{BE}$?
   They are congruent because they are radii of the same circle by construction.

Getting an Answer

6. Write a paragraph proof that uses corresponding parts of congruent triangles to prove that $\angle A \equiv \angle B$.
   $\overline{AC} \equiv \overline{BE}$ and $\overline{AD} \equiv \overline{BF}$ because they are both radii of the same circle. $\overline{DC} \equiv \overline{FE}$ because they both have the same compass settings. Therefore, $\triangle ACD \equiv \triangle BFE$ by SSS and $\angle A \equiv \angle B$ by CPCTC.
## Additional Vocabulary Support
### Isosceles and Equilateral Triangles

Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>base</strong></td>
<td>The base of an isosceles triangle is the side included between the pair of congruent angles.</td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>base angles</strong></td>
<td>1. The base angles are the two congruent angles of an isosceles triangle.</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>corollary</strong></td>
<td>2. A corollary is a theorem that can be proved easily by another theorem.</td>
<td>A corollary to the Isosceles Triangle Theorem is: If a triangle is equilateral, then the triangle is equiangular.</td>
</tr>
<tr>
<td><strong>equiangular triangle</strong></td>
<td>An equiangular triangle is a triangle with three congruent angles. The angles of an equiangular triangle all measure 60.</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>isosceles triangle</strong></td>
<td>4. An isosceles triangle is a triangle with two congruent sides.</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>legs</strong></td>
<td>The legs are the two congruent sides of an isosceles triangle.</td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>vertex angle</strong></td>
<td>6. The vertex angle is the angle formed by the legs of an isosceles triangle.</td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
</tbody>
</table>
4-5 Reteaching (continued)

Isosceles and Equilateral Triangles

**Problem**

What is the value of \( x \)?

Because \( x \) is the measure of an angle in an equilateral triangle, \( x = 60 \).

**Problem**

What is the value of \( y \)?

\[
m\angle DCE + m\angle DEC + m\angle EDC = 180
\]

\[
60 + 70 + y = 180
\]

\[
y = 50
\]

**Exercises**

Complete each statement. Explain why it is true.

1. \( \angle EAB \equiv \boxed{\square} \) \( \angle EBA \); base angles of an isosceles triangle are congruent.
2. \( \angle BCD \equiv \boxed{\square} \equiv \angle DBC \)
   \( \angle CDB \); the angles of an equilateral triangle are congruent.
3. \( \overline{FG} \equiv \boxed{\square} \equiv \overline{DF} \)
   \( \overline{GD} \); the sides of an equilateral triangle are congruent.

Determine the measure of the indicated angle.

4. \( \angle ACB \ 60 \)
5. \( \angle DCE \ 65 \)
6. \( \angle BCD \ 55 \)

**Algebra** Find the value of \( x \) and \( y \).

7. \( \boxed{35; 35} \)
8. \( \boxed{65; 50} \)

9. **Reasoning** An exterior angle of an isosceles triangle has a measure 140.
   Find two possible sets of measures for the angles of the triangle. \( 40, 40, 100; 40, 70, 70 \)
4-5 Think About a Plan
Isosceles and Equilateral Triangles

**Algebra** The length of the base of an isosceles triangle is \( x \). The length of a leg is \( 2x - 5 \). The perimeter of the triangle is 20. Find \( x \).

**Know**
1. What is the perimeter of a triangle?
   - the sum of the sides of a triangle

2. What is an isosceles triangle?
   - a triangle with two sides the same length

**Need**
3. What are the sides of an isosceles triangle called?
   - base and leg

4. How many of each type of side are there?
   - 1 base and 2 legs

5. The lengths of the base and one leg are given. What is the third side of the triangle called?
   - leg

**Plan**
6. Write an expression for the length of the third side. \( 2x - 5 \)

7. Write an equation for the perimeter of this isosceles triangle.
   \( x + (2x - 5) + (2x - 5) = 20 \)

8. Solve the equation for \( x \). Show your work.
   \[
   \begin{align*}
   5x - 10 &= 20 \\
   5x &= 30 \\
   x &= 6
   \end{align*}
   \]
Additional Vocabulary Support

Concordance in Right Triangles

The column on the left shows the steps used to prove that $\overline{AB} \cong \overline{ED}$. Use the column on the left to answer each question in the column on the right.

<table>
<thead>
<tr>
<th>Problem</th>
<th>$\cong$ in Right Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given: $C$ is the midpoint of $\overline{AE}$ and $\overline{BD}$.</td>
<td></td>
</tr>
<tr>
<td>Prove: $\overline{AB} \cong \overline{ED}$</td>
<td></td>
</tr>
</tbody>
</table>

1. What is the definition of the midpoint of a line segment?
   The midpoint is halfway between the two endpoints. It divides the line segment into two equal halves.

2. How do you know that $\overline{AB} \parallel \overline{DE}$ and $m\angle B = 90$?
   by reading the diagram

3. What does the symbol $\cong$ between two line segments mean?
   The line segments are congruent.

4. What does the word *interior* mean?
   Interior means on the inside of.

5. What is the measure of $\angle D$?
   90

6. What information is necessary to apply the HL Postulate?
   Two right triangles must have congruent hypotenuses and one congruent leg.

7. What are the corresponding angles and sides for $\triangle ABC$ and $\triangle EDC$?
   $\angle B$ and $\angle D$, $\angle A$ and $\angle E$, $\angle ABC$ and $\angle DCE$, $\overline{AB}$ and $\overline{ED}$, $\overline{AC}$ and $\overline{EC}$, $\overline{BC}$ and $\overline{DC}$
Exercises

Determine if the given triangles are congruent by the Hypotenuse-Leg Theorem. If so, write the triangle congruence statement.

1. \( \triangle TUV \) is not congruent.

2. \( \triangle RSZ \cong \triangle TSZ \)

3. \( \triangle LMN \cong \triangle RVS \)

Measure the hypotenuse and length of the legs of the given triangles with a ruler to determine if the triangles are congruent. If so, write the triangle congruence statement.

4. \( \triangle OPQ \cong \triangle ZYX \)

5. \( \triangle ABC \cong \triangle CMA \)

6. \( \triangle EFG \cong \triangle HU \)

7. Explain why \( \triangle LMN \cong \triangle OMN \). Use the Hypotenuse-Leg Theorem. Because \( \angle NML \) and \( \angle NMO \) are right angles, both triangles are right triangles. It is given that their hypotenuses are congruent. Because they share a leg, one pair of corresponding legs is congruent. All criteria are met for the triangles to be congruent by the Hypotenuse-Leg Theorem.

8. Visualize \( \triangle ABC \) and \( \triangle DEF \), where \( AB = EF \) and \( CA = FD \). What else must be true about these two triangles to prove that the triangles are congruent using the Hypotenuse-Leg Theorem? Write a congruence statement. \( \angle B \) and \( \angle E \) are right angles, or \( \angle C \) and \( \angle D \) are right angles. \( \triangle ABC \cong \triangle DEF \) or \( \triangle ABC \cong \triangle FED \).
4-6  Think About a Plan
Congruence in Right Triangles

Algebra  For what values of \( x \) and \( y \) are the triangles congruent by HL?

Know
1. For two triangles to be congruent by the Hypotenuse-Leg Theorem, there must be a pair of right angles, and the lengths of the hypotenuses and one of the legs of each triangle must be equal.

2. The length of the hypotenuse of the triangle on the left is \( 3y + x \) and the hypotenuse of the triangle on the right is \( y + 5 \).

3. The length of the leg of the triangle on the left is \( y - x \) and the length of the leg of the triangle on the right is \( x + 5 \).

Need
4. To solve the problem you need to find the values of \( x \) and \( y \).

Plan
5. What system of equations can you use to find the values of \( x \) and \( y \)?
\[
3y + x = y + 5; y - x = x + 5
\]

6. What method(s) can you use to solve the system of equations?
Sample: Solve one equation for \( y \) and substitute into the other equation; graph both equations.

7. What is the value of \( y \)? What is the value of \( x \)? \( 3; -1 \)
**4-7 Additional Vocabulary Support**

**Congruence in Overlapping Triangles**

A student wanted to prove $\overline{EB} \cong \overline{DB}$ given $\angle AED \cong \angle CDE$ and $\overline{AE} \cong \overline{CD}$. She wrote the statements and reasons on note cards, but they got mixed up.

**Use the note cards to write the steps in order.**

1. First, $\triangle AED \cong \triangle CDE$ and $\overline{AE} \cong \overline{CD}$ are given.

2. Second, $\overline{ED} \cong \overline{DE}$ by the Reflexive Property of Congruence.

3. Third, $\triangle AED \cong \triangle CDE$ by the Side-Angle-Side (SAS) Theorem.

4. Next, $\angle A \cong \angle C$ because corresponding parts of congruent triangles are congruent.

5. Then, $\angle ABE \cong \angle CBD$ because vertical angles are congruent.

6. Then, $\triangle ABE \cong \triangle CBD$ by the Angle-Angle-Side (AAS) Theorem.

7. Finally, $\overline{EB} \cong \overline{DB}$ because corresponding parts of congruent triangles are congruent.
Reteaching (continued)

Congruence in Overlapping Triangles

Separate and redraw the overlapping triangles. Identify the vertices.

1. \( \triangle GLJ \) and \( \triangle HIL \)

2. \( \triangle MRP \) and \( \triangle NQS \)

3. \( \triangle FED \) and \( \triangle CDE \)

Fill in the blanks for the two-column proof.

4. Given: \( \angle AEG \equiv \angle AFD, \overline{AE} \equiv \overline{AF}, \overline{GE} \equiv \overline{FD} \)
   Prove: \( \triangle AFG \equiv \triangle AED \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \angle AEG \equiv \angle AFD, \overline{AE} \equiv \overline{AF}, \overline{GE} \equiv \overline{FD} )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle AEG \equiv \angle AFD )</td>
<td>2) SAS</td>
</tr>
<tr>
<td>3) ( \overline{AG} \equiv \overline{AD}, \angle G \equiv \angle D )</td>
<td>3) CPCTC</td>
</tr>
<tr>
<td>4) ( \overline{GE} \equiv \overline{FD} )</td>
<td>4) Given</td>
</tr>
<tr>
<td>5) ( \overline{GE} \equiv \overline{FD} )</td>
<td>5) Def. of ( \equiv )</td>
</tr>
<tr>
<td>6) ( GF + FE = GE, FE + ED = FD )</td>
<td>6) Seg. Addition Post.</td>
</tr>
<tr>
<td>7) ( GF + FE = FE + ED )</td>
<td>7) Substitution Property</td>
</tr>
<tr>
<td>8) ( GF = ED )</td>
<td>8) Subtr. Prop. of Equality</td>
</tr>
<tr>
<td>9) ( \triangle AFG \equiv \triangle AED )</td>
<td>9) SAS</td>
</tr>
</tbody>
</table>

Use the plan to write a two-column proof.

5. Given: \( \angle PSR \) and \( \angle PQR \) are right angles, \( \angle QPR \equiv \angle SRP \).
   Prove: \( \triangle STR \equiv \triangle QTP \)

Plan for Proof:
Prove \( \triangle QPR \equiv \triangle SRP \) by AAS. Then use CPCTC and vertical angles to prove \( \triangle STR \equiv \triangle QTP \) by AAS.
4-7 Think About a Plan
Congruence in Overlapping Triangles

Given: \( QT \perp PR, QT \text{ bisects } PR, QT \text{ bisects } VQS \)
Prove: \( VQ \equiv SQ \)

Know
1. What information are you given? What else can you determine from the given information and the diagram?
   \( QT \perp PR, QT \text{ bisects } PR, QT \text{ bisects } VQS; \angle PQT \text{ and } \angle RQT \text{ are right angles}, PQ = QR, \angle VQT = \angle SQT \)

2. To solve the problem, what will you need to prove?
   \( VQ \equiv SQ \)

Need
3. For which two triangles are \( VQ \) and \( SQ \) corresponding parts?
   \( \triangle PQV \text{ and } \triangle RQS \) or \( \triangle VQT \text{ and } \triangle SQT \)

4. You need to use corresponding parts to prove the triangles from Exercise 3 congruent. Which two triangles should you prove congruent first, using the given information? Which theorem or postulate should you use?
   \( \triangle PQT \text{ and } \triangle RQT \text{ by SAS} \)

5. Which corresponding parts should you then use to prove that the triangles in Exercise 3 are congruent?
   Answers may vary. Sample: \( \angle P \) and \( \angle R \)

Plan
6. Use the space below to write the proof.
   Statements: 1) \( QT \perp PR; QT \text{ bisects } PR, QT \text{ bisects } VQS; 2) m\angle PQT = m\angle RQT = 90; 3) PQ = QR; 4) QT = QT; 5) \triangle PQT \equiv \triangle RQT; 6) \angle P \equiv \angle R; 7) \triangle PQV \text{ and } \triangle VQT \text{ are compl}; \angle RQS \text{ and } \angle SQT \text{ are compl}; 8) \angle VQT \equiv \angle SQT; 9) \triangle PQV \equiv \triangle RQS; 10) \triangle PQV \equiv \triangle RQS; 11) VQ \equiv SQ; Reasons: 1) Given; 2) Definition of perpendicular lines; 3) Definition of bisector; 4) Reflexive Property of Congruence; 5) SAS; 6) CPCTC; 7) Definition of complementary angles; 8) Definition of angle bisector; 9) Complements of are \( \equiv \); 10) ASA; 11) CPCTC